An Optimized Design of Network Arch Bridge using Global Optimization Algorithm

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Abstract: An optimization approach of network arch bridges using a global optimization algorithm EVOP is presented in this paper. The objective is to minimize the cost of superstructure particularly arches and hangers of network arch bridge by optimizing the geometric shape, rise to span ratio, cross section of arch, and the number, arrangement and cross sectional dimensions of hangers. Constraints for design are formulated as per AASHTO, AISC and ACI Specifications. The minimum cost design problem is characterized by having a combination of continuous, discrete, and integer sets of design variables and is subjected to highly nonlinear, implicit and discontinuous constraints. An optimization algorithm, evolutionary operation (EVOP), is used that is capable of locating directly with high probability the global minimum without requiring information on gradient or sub-gradient of the objective function. EVOP is interfaced with finite element analysis software, ANSYS for evaluation of structural response of the bridge to verify the constraints and also to evaluate objective function. Within the range of design constant parameters considered, it is observed that 38% to 40% of total cost can be saved for circular and parabolic arches respectively, if design is optimized. Results also show that parabolic arch with optimum design variables is more economic than the optimized arch bridges with circular arch geometry.

Key words: global optimization, EVOP, arch bridges, structural optimization, finite element analysis.

1. INTRODUCTION

Network arch bridges seem to be very competitive for road bridges of spans of 135 m to 160 m, due to its beneficial structural behavior which is mainly subjected to axial forces (Tveit 2003). Furthermore, the high stiffness and therefore small deflections favor the application of this kind of bridges for high speed railway as well as roadway transportations. Research on optimizing realistically three-dimensional structures especially large structures, is now feasible due to advances in numerical optimization methods, computer based numerical tools for analysis of structures and availability of powerful computing hardware. Thus, large and important projects like network arch bridges have the potential for substantial cost reduction through application of optimum design methodology.

A few researches regarding optimization of network arch bridges have been performed in recent years. Tveit (1980) developed some concepts regarding hanger arrangement to reduce bending moments in arch and also in hangers. Brunn *et al.* (2003) adopted a hypothesis for optimization of hanger arrangement that arch should be a part of a circle and hanger should be arranged in such a way that hanger intersections lie on the radii of the arch circle. They adopted trial and error based traditional method for optimization of various

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design parameters. Again, Tveit (2008) suggested that optimized network arches can be achieved if some hangers cross other hangers at least twice.

Previous works on optimization of the network arch bridges with inclined hangers has been adopted based on some hypothesis and followed traditional method of design. One of the main limitations of traditional optimization approaches is that they can easily be entrapped in local minima i.e. best possible solution may not be achieved. As a result, traditional approaches have difficulties to determine the global optimum. Global optimization of network arch bridge system considering all possible design variables has not yet been performed. In this study, simulation driven optimization of the present bridge system is performed by adopting an evolutionary based global optimization method, EVOP developed by Ghani (1989) which has high probability for seeking best possible solution.

2. OPTIMAL DESIGN PROBLEM STATEMENT

In this study, the objective of design is cost minimization of superstructure particularly arch and hangers of network arch bridges by taking into account costs of materials, fabrication and installation. Unit costs used in this study are based on Roads and Highway Department cost schedule (RHD 2008).Total cost is determined as:

$$C_T = C_{HC} + C_{AC} + C_{AS} \quad (1)$$

where, C_{HC}, C_{AC} and C_{AS} are the costs of cable of hangers, concrete section of arch and amount of reinforcement required all over the arch respectively. Costs of individual components are calculated as:

$$C_{HC} = UP_{HC}W_{HC} \quad (2)$$

$$C_{AC} = UP_{AC}V_{AC} \quad (3)$$

$$C_{AS} = UP_{AS}W_{AS} \quad (4)$$

where, UP_{HC}, UP_{AC} and UP_{AS} are the unit prices of materials, fabrication and installation of hangers, concrete of arch and the reinforcement required in the arch respectively; W_{HC}, V_{AC} and W_{AS} are the weight of the cables, volume of the concrete required in arches and weight of reinforcement required in arches respectively.

Design variables with explicit constraints, design constant parameters and implicit constraints for the optimization problem are enlisted in Tables 1–3, where upper and lower bounds of constraints come from ACI (2002), AASHTO (2002) and AISC (2005) design considerations. Design variables and design constant parameters are specifically shown in Figures 1–6.

3. OPTIMIZATION PROCESS

Nine numbers of design variables and seven numbers of implicit constraints are associated with the bridge optimization problem under discussion. The design variables are a combination of continuous, discrete and integer types. The minimum cost design problem is subjected to highly nonlinear, implicit and discontinuous constraint having multiple local minima which requires an optimization method to derive the global optimum. To deal with this optimization problem, the global optimization algorithm named EVOP is used. It has the capability to locate directly with high probability the global minimum. It has the ability to minimize directly an objective function without requiring information on gradient or sub-gradient. There is no requirement for scaling of objective and constraining functions. The method is good to minimize functions of mixed continuous, discrete and integer variables. It is ideally suited for system optimization of real life design. It does not require training and initial known population size as required by genetic algorithm. Objective and constraining functions can possess finite number of discontinuities. It has facility for automatic restarts to check whether the previously obtained minimum is the global minimum. EVOP was extensively tested on more than fifty test functions well known for their numerical difficulties and resistance to locating the minimum. Detail and nature of these test functions are available in Ghani (2008).

The problem formulation is as follows.

$$\text{Find design variables, } x_k = \left\{ x_1, x_2, x_3 \text{..........} x_{NDIV} \right\} \quad (5)$$

to minimize the objective function, $F(x)$

subjected to implicit constraints,

$$G(x)_i^L \le G(x)_i \le G(x)_i^U \quad (6)$$

with explicit constraints on

$$\text{design variables, } x_k^L \le x_k \le x_k^U \quad (7)$$

where $i = 1, 2\dots\dots NIC$; $k = 1, 2\dots\dots NDIV$; NIC is total number of implicit constraints; $NDIV$ is total

Table 1. Design variables with explicit constraints

Design Variables	Variable Type	Explicit Constraint	
		Lower bound	Upper Bound
Number of Hangers (Each Arch), N_h	Integer	4	60
Start Angle for Hanger Inclination Set1, $\varphi 1$	Continuous	0°	80°
Start Angle for Hanger Inclination Set2, $\varphi 2$	Continuous	0°	80°
Angle Change for $\varphi 1$, $\Delta\varphi 1$	Continuous	–2°	2°
Angle Change for $\varphi 2$, $\Delta\varphi 2$	Continuous	–2°	2°
Cross Sectional Area of Cable of Hanger, A_h	Discrete	96.8 mm^2	2929 mm^2
Arch Width, B_h	Discrete	250 mm	3000 mm
Arch Depth, H_h	Discrete	250 mm	4000 mm
Rise to Span Ratio, R_h	Continuous	0.14	0.25

Table 2. Design constant parameters

	Design Constant Parameters	Value
Material Properties	Modulus of Elasticity for Concrete, E_C	24800 MPa
	Poisson's Ratio of Concrete, υ	0.2
	Concrete Compressive Strength at 28 days (Arch and Bracing), f_{ca}'	25 MPa
	Concrete Compressive Strength at 28 days (Deck), f_{cd}'	50 MPa
	Ultimate Strength of Cable of Hanger, f_u	1520 MPa
	Arch Reinforcement Yield Stress, f_y	413 MPa
Geometric Properties	Span, L	100 m
	Width of Bridge, B_w	10 m
	Arch Section	Rectangular
	Arch Shape	Circular, Parabolic
Loading	Standard Vehicle Load	AASHTO HS 20-44 Single and Lane
	No of Lane	Two
	Wearing Surface	1436 N/m^2
Cost Parameters	Unit Price for Cable of Hanger, UP_{HC}	223 BDT/kg
	Unit Price for Concrete of Arch, UP_{AC}	10527 BDT/m^3
	Unit Price for Reinforcement of Arch, UP_{AS}	80 BDT/kg
General	Design Code	AASHTO (2002), AISC (2005)

Table 3. Implicit constraints

Response	Implicit Constraint	
	Lower bound	Upper Bound
Extreme Hanger Stress, σ_{max}	0	0.75 F_u or 0.9 Fy
Strength Criteria of Arch, *CRT*	0	1
Design Reinforcement Factor in Arch, *RNR*	1%	8%
End Angle for Hanger Inclination Set1, $\varphi 1_n$	–80°	80°
End Angle for Hanger Inclination Set2, $\varphi 2_n$	–80°	80°
In Plane Slenderness ratio	0.1	22
Arch Depth/ Width Ratio	0.25	6

number of independent design variables. $G(x)_i$ and $F(x)$ are implicit constraints and objective function respectively. The functions $G(x)_i$ and $F(x)$ have been evaluated from finite element simulation in the present study. The lower and upper bound of explicit constraints and implicit constraints may be either constants or functions of independent variables. The implicit constraints are allowed to make the feasible vector-space non-convex.

There are six fundamental processes in the EVOP,

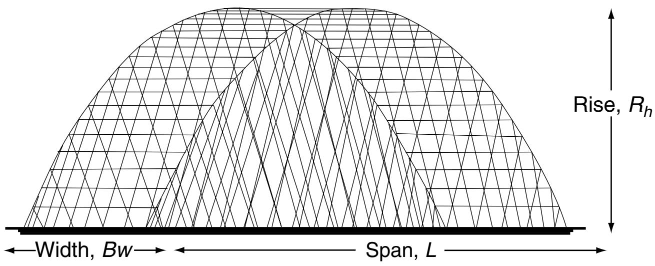

Figure 1. 3D view of network arch bridge

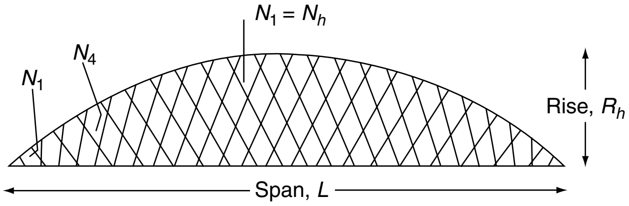

Figure 2. Typical hanger arrangement

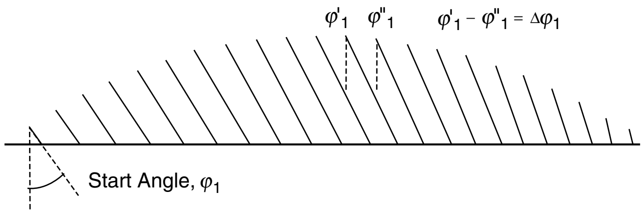

Figure 3. Hanger set 1 and its vertical inclination

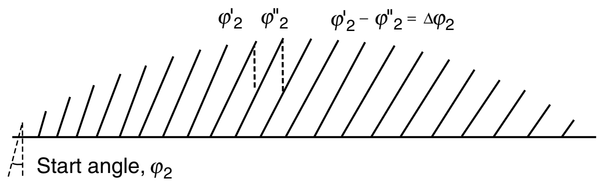

Figure 4. Hanger set 2 and its vertical inclination

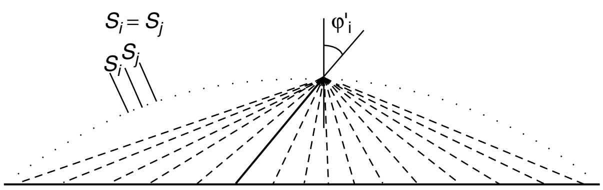

Figure 5. Alternate hanger position from one of equidistant hanger nodes along the arch

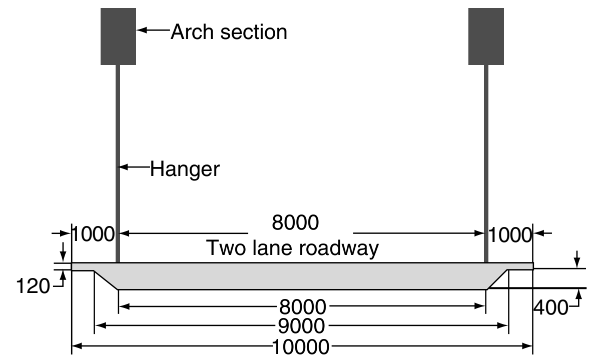

Figure 6. Deck section (Dimension in mm)

viz. Generation of a 'complex', Selection of a 'complex' vertex for penalization, Testing for collapse of a 'complex', Dealing with a collapsed 'complex', Movement of a 'complex' and Convergence tests. Detail description of the six processes can be found in Ghani (1989). To understand the algorithm, some information regarding "complex" is described here. A 'complex' is an object which occupies an N-dimensional parameter space defined by $K \geq (N + 1)$ vertices inside a feasible region where, N is the number of design variables. It can rapidly change its shape and size for negotiating difficult terrain and has the intelligence to move towards a minimum. Figure 7 (Rana *et al.* 2013) shows a 'complex' with four vertices in a two dimensional parameter space (X_1 and X_2 axes). X_1 and X_2 are the two independent design variables. The 'complex' vertices are identified by lower case letters 'a', 'b', 'c' and 'd' in an ascending order of function values, i.e. F(a) < F(b) < F(c) < F(d). Each of the vertices has two co-ordinates-(X_1, X_2). Straight line parallel to the co-ordinate axes are explicit constraints with fixed upper and lower limits. The curved lines represent implicit constraints set to either upper or lower limits. The hatched area is the two dimensional feasible region. The general outline of EVOP algorithm is illustrated in Figure 8 (Ahsan *et al.* 2012).

At first, all $(k - 1)$ vertices of the complex that satisfy all explicit and all implicit constraints are randomly generated beginning from a single feasible starting

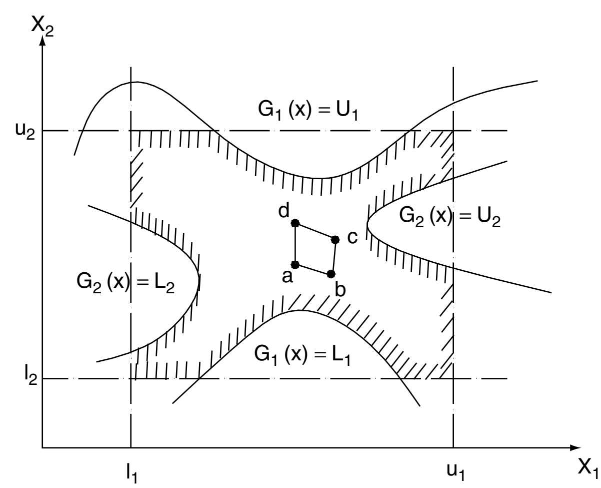

Figure 7. A "complex" with four vertices

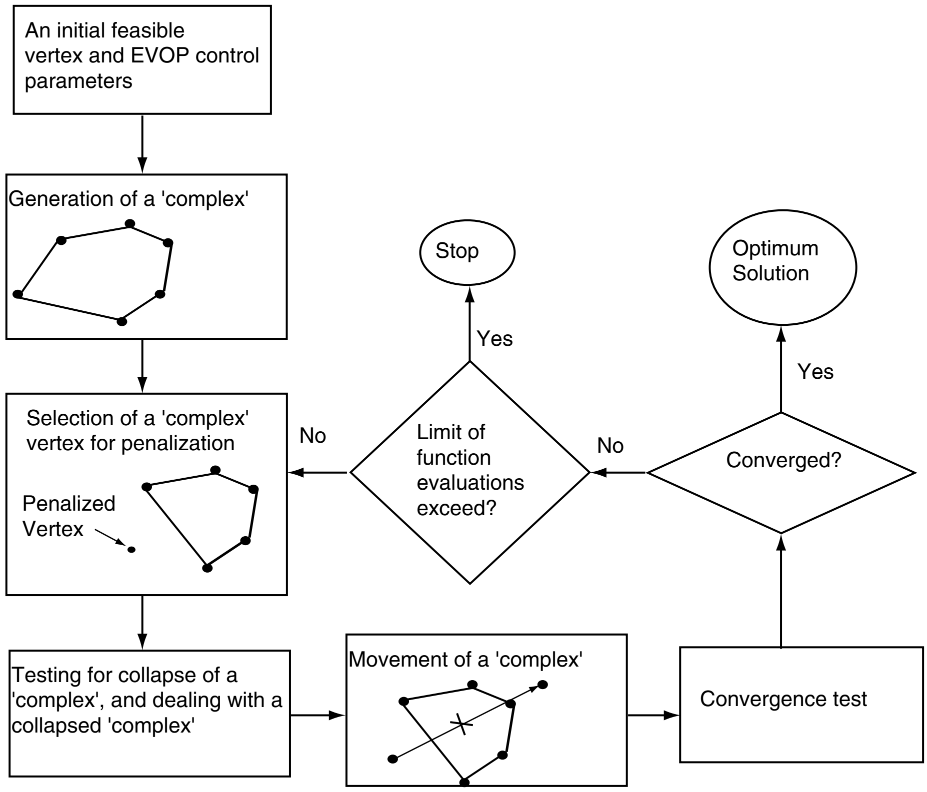

Figure 8. General outline of EVOP algorithm

point. Next, the worst vertex of the 'complex' (the vertex with the highest function value) is penalized by over-reflecting on the centroid of the complex. For this penalization to be successful, testing for collapse of the 'complex' is performed. A 'complex' is said to have collapsed in a subspace if the i^{th} coordinate of the centroid is identical to the same of all 'k' vertices of the 'complex'. This is a sufficiency condition and detects collapse of a 'complex' when it lies parallel along a coordinate axis. Once a 'complex' has collapsed in a subspace it will never again be able to span the original N-dimensional space. After detecting the collapse of a 'complex' on to a subspace some actions are taken such that a new full sized 'complex' is generated. It now spans the n-dimensional feasible space as defined by the explicit and implicit constraints. The 'complex' is next moved. The movement of complex process begins by over-reflecting the worst vertex of a 'complex' on the feasible centroid of the remaining vertices to generate a new trial point. If the refection step is unsuccessful, then some contraction steps and expansion step are applied sequentially to generate a new feasible trial point using the contraction coefficient (β) and expansion coefficient (γ) respectively. However, before penalizing a worst vertex the feasibility of the centroid of the remaining (k–1) vertices is checked. If it is infeasible appropriate steps are taken to regain feasibility. While executing the process of movement of a 'complex', tests for convergence are made periodically after certain preset number of calls to the objective function.

To interface the finite element simulation with evolutionary operation, EVOP which is originally written in FORTRAN, a platform is established by visual *C*++. The platform is used to transfer the parameters from EVOP to the simulator input file and to extract the response values of interest from the simulator's output file for return to EVOP. In the whole optimization process, the platform in visual *C*++, EVOP in FORTRAN and ANSYS (2009) are interlinked in the process as shown in Figure 9.

Figure 10 shows optimization flow chart denoting file input output operations. An initial feasible design determined from conventional design of the system which satisfies all implicit and explicit constraints, is invoked through the interfacing platform which runs EVOP and acts as platform for data structure definition. Design variables are intelligently allocated by EVOP. EVOP transfers the parameters to user's simulation code through the interfacing platform. Simulation output file provides necessary information for EVOP through platform and updated design variables are dynamically decided by the optimization platform for new simulation. The process is repeated until all of the

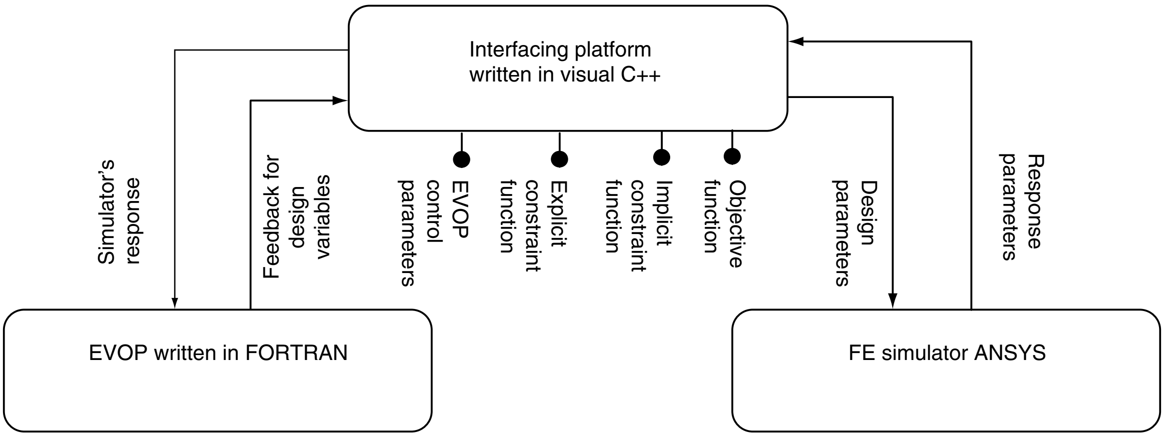

Figure 9. Black box interfacing

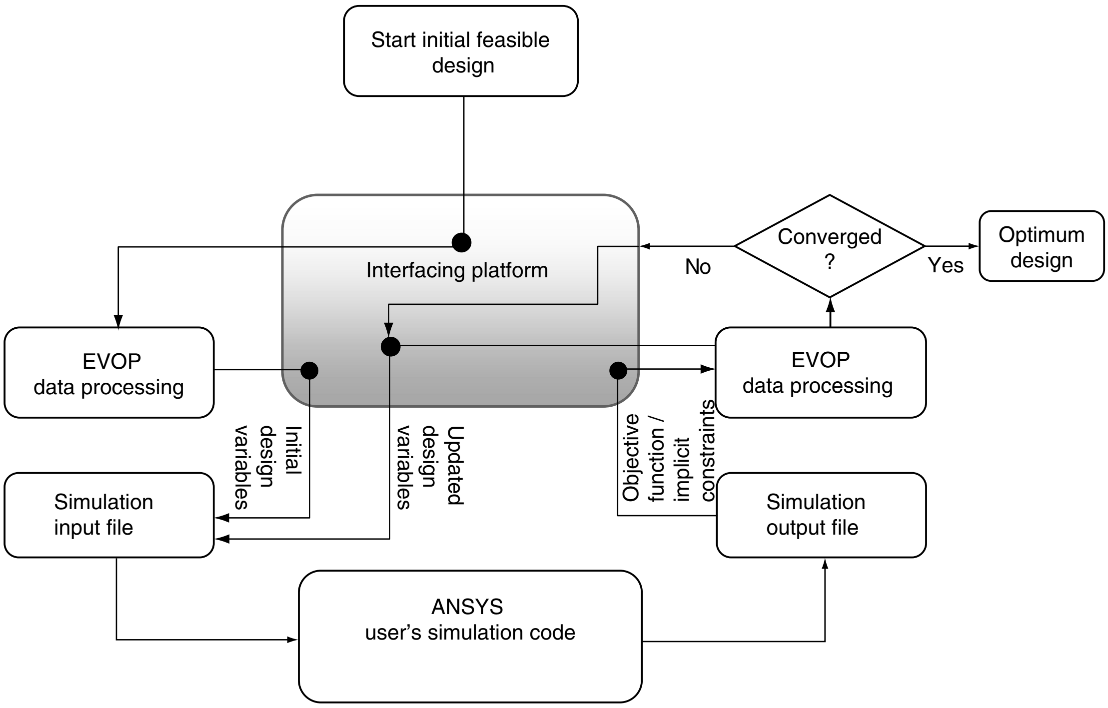

Figure 10. Optimization flow chart

simulation code runs required by the iterative study have been completed. The solution is then said to be converged and optimum results are recorded. A benchmark problem of Prasad and Haftka (1979) in Verification Manual of ANSYS "Shape Optimization of a Cantilever Beam" is solved to check the EVOP optimization process. Result in (Islam 2010) draws successful verification of interfacing of EVOP with finite element software, ANSYS on benchmark optimization problems.

4. FINITE ELEMENT ANALYSIS OF NETWORK ARCH BRIDGES

The complete 3D finite element model of the bridge has been introduced by assigning appropriate element types and geometries, meshing, assigning proper boundary and load conditions. ANSYS is chosen for finite element simulation of the arch bridge. Beam, link and solid elements have been used for the simulation of arch, hanger and deck of the bridge respectively. Surface element is incorporated on deck nodes for 3D structural surface effect. A load family is assigned for the highway load by the vehicle type AASHTO standard HS 20–44 truck and lane load. The complete finite element model of the bridge is shown in Figures 11 and 12.

It has been considered that arch and arch bracing are to be made of reinforced concrete, hanger is made of zinc-coated steel structural strand of ASTM A586 (1998) standard and deck is of reinforced concrete. In the finite element modeling deck is introduced for

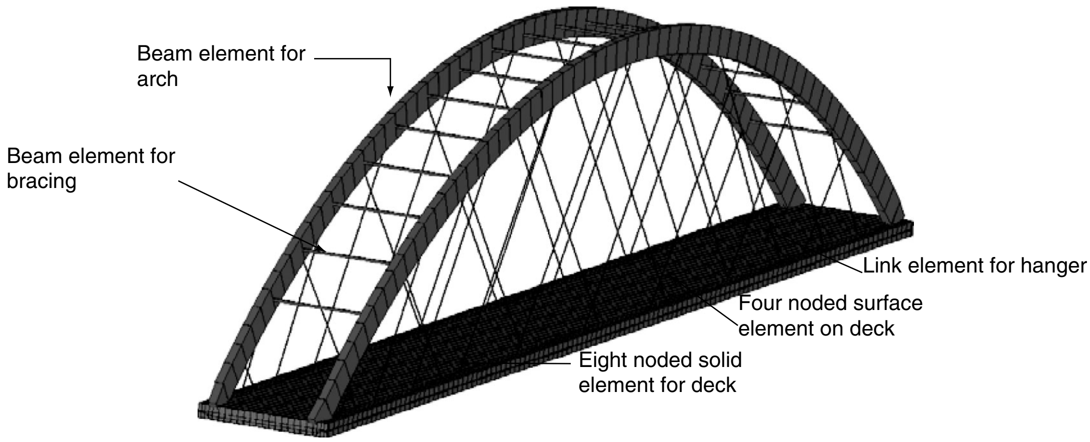

Figure 11. Typical finite element model of network arch bridges

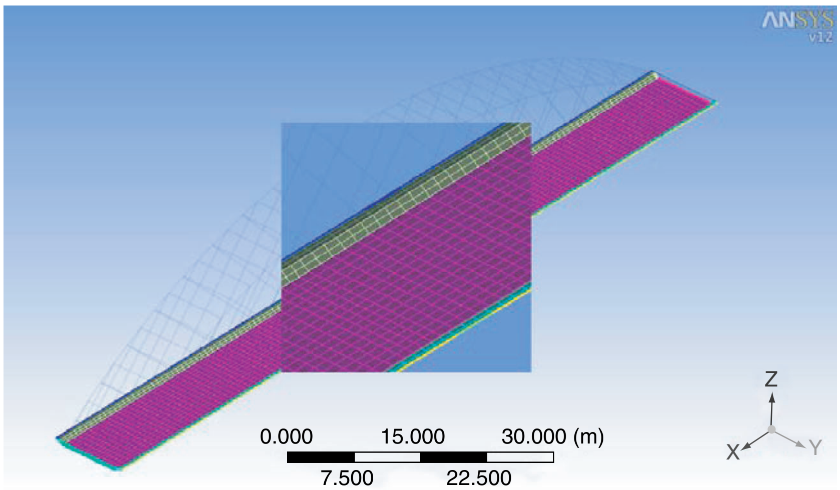

Figure 12. Finite element mesh in deck

vehicle movement and to transfer load to arch through hanger. Material properties of reinforced concrete and cable are shown in Table 4. Vehicle movement and response of the bridge are shown in Figures 13–15.

After performing finite element analysis of network arch bridges, ANSYS CivilFEM has been used for structural evaluation and then the response is utilized by the optimization algorithm for further data processing. Scripts have been written to record maximum hanger stress of all available hangers for all possible load combinations and considering all vehicle positions acting on the deck. Similarly maximum percentage of steel required in each arch element and total enveloped reinforcement, for all possible load combinations and considering all vehicle positions acting on the deck, is recorded following design of arch using CivilFEM. From post processing consideration, two different scripts of FE analysis and design are written and results from analysis are post processed subsequently which is available for object function and implicit function of EVOP. A series of function evaluation is required by the optimization algorithm, EVOP and EVOP guides finite element software for analysis of the structure by batch execution and terminate after each data processing. Total iteration of FE analysis is completed when the result of optimization is converged to its optimum design. A high end machine has been used in this study for reducing computational time in the optimization process of fine element model of network arch bridges. The convergence of the optimization algorithm was found fast. Two-three restart of the algorithm is found enough for the convergence. In addition, in each restart of EVOP 100–500 FE analysis is carried out by the algorithm depending on the initial starting point.

5. RESULTS OF OPTIMIZATION

Optimum design parameters for circular and parabolic arch bridges compared with initial design is shown in Figures 16 and 17 and in Table 5. Influence lines of

Table 4. Material properties

Material	Parameters	Value
Concrete	Modulus of Elasticity for Concrete, E_c	24800 MPa
	Poisson's Ratio, υ	0.2
	Concrete Compressive Strength at 28 days (Arch and Bracing)	25 MPa
	Concrete Compressive Strength at 28 days (Deck)	50 MPa
	Unit Weight	24.5 N/m^3
Reinforcement	Arch Reinforcement Yield Stress, f_y	413 MPa
	Modulus of Elasticity, E_s	200×10^3 MPa
	Poisson's Ratio, υ	0.3
	Density, ρ	7833 kg/m^3
Cable of Hanger	Ultimate Strength of Cable of Hanger	1520 MPa
	Modulus of Elasticity for Concrete, E_c	195×10^3 MPa
	Poisson's Ratio, υ	0.3
	Density, ρ	8000 kg/m^3
Surface Elements	Density, ρ	0.0 kg/m^3

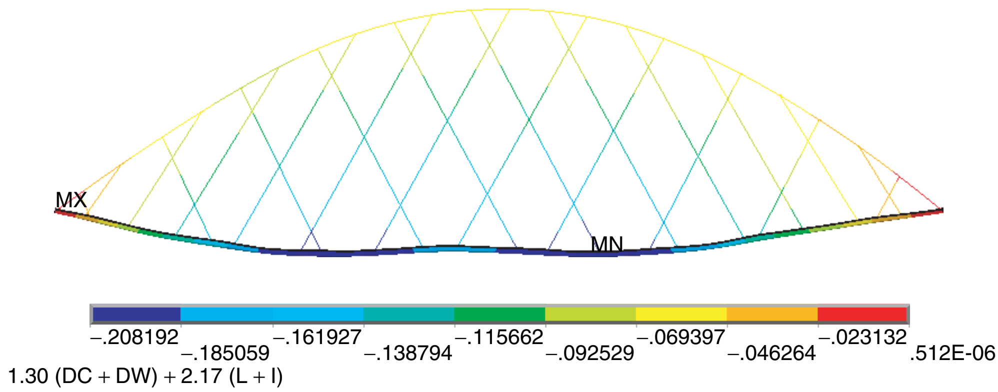

Figure 13. Typical deflection for dead and live load

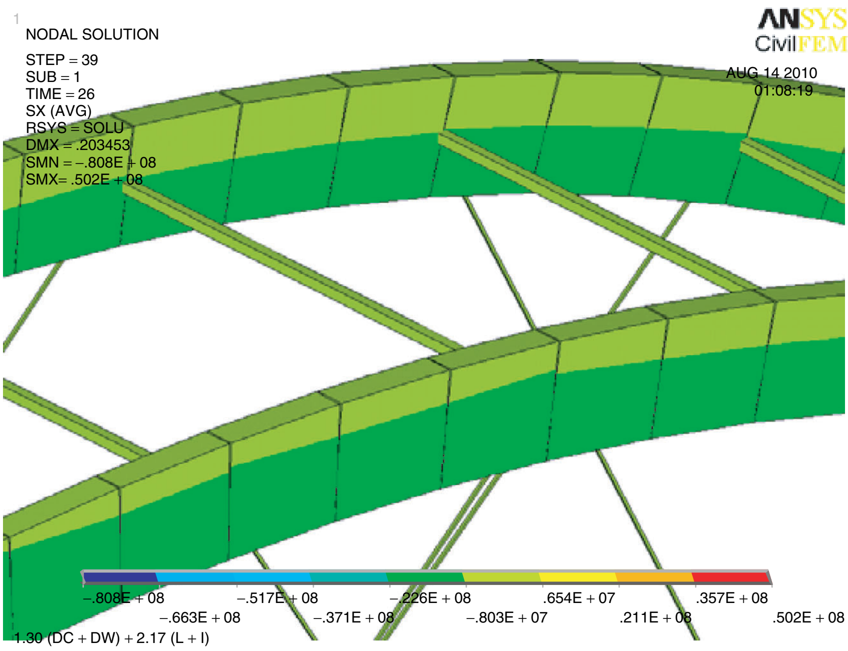

Figure 14. Typical stress contour for dead and live load

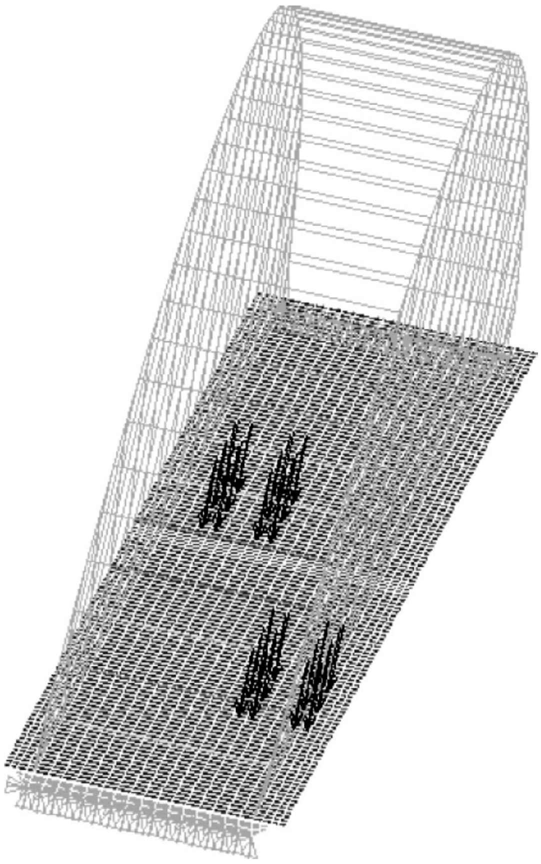

Figure 15. Typical multi load step for AASHTO HS 20-44 truck load

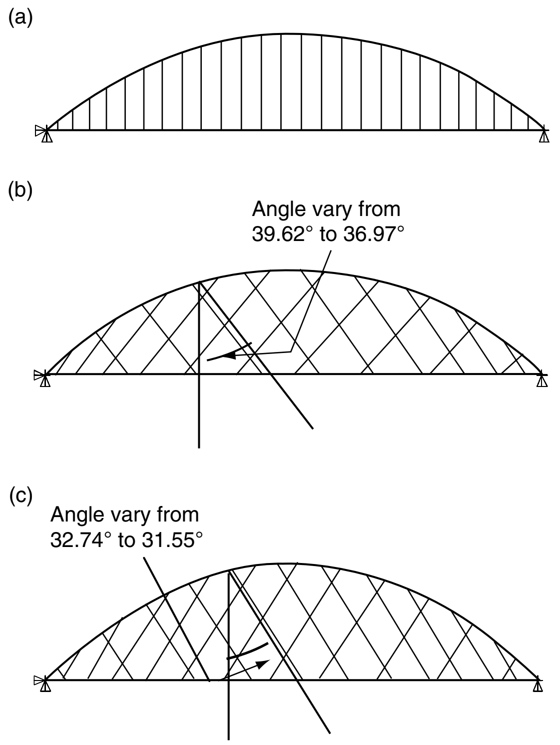

Figure 16. Arch bridge with: (a) initial design variables; (b) optimum design variables for circular arch; (c) optimum design variables for parabolic arch

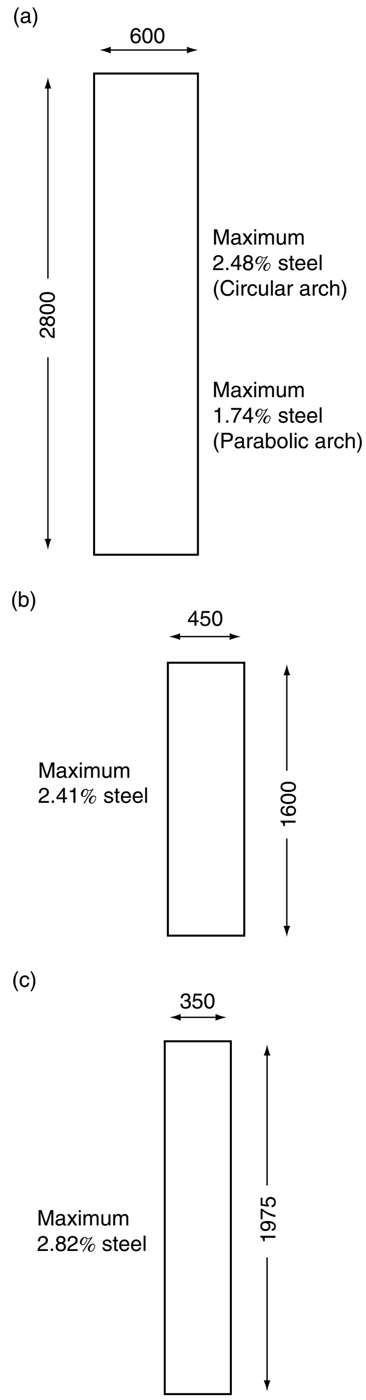

Figure 17. Arch section: (a) before optimization; (b) after optimization for circular arch; (c) after optimization for parabolic arch (all dimensions are in mm)

bending moment for optimized arrangements and vertical hanger arrangements with the same arch section are compared in Figure 18 for circular arch geometry.

Ratio of influence line ordinate for bending at different location is depicted in Figure 19 where ratio of influence line ordinate is defined as the influence line ordinate for vertical hanger

Table 5. Design variables for before and after optimization

	Parameter	Unit	Before optimization	After optimization (Circular)	After optimization (Parabolic)
Design Variables	Number of Hangers, N_h	—	26	20	22
	Start Angle for Hanger Inclination Set1, $\varphi 1$	Degree	0	39.62	32.74
	Start Angle for Hanger Inclination Set2, $\varphi 2$	Degree	0	36.97	31.55
	Angle Change for $\varphi 1$, $\Delta\varphi 1$	Degree	0	–0.265	–0.11
	Angle Change for $\varphi 2$, $\Delta\varphi 2$	Degree	0	0.265	0.11
	Cross Sectional Area of Cable of Hanger, A_h	mm^2	800	1548.4	1025.8
	Arch Width, B_h	mm	600	450	350
	Arch Depth, H_a	mm	2800	1600	1975
	Rise to Span Ratio, R_h	—	0.18	0.2105	0.2386

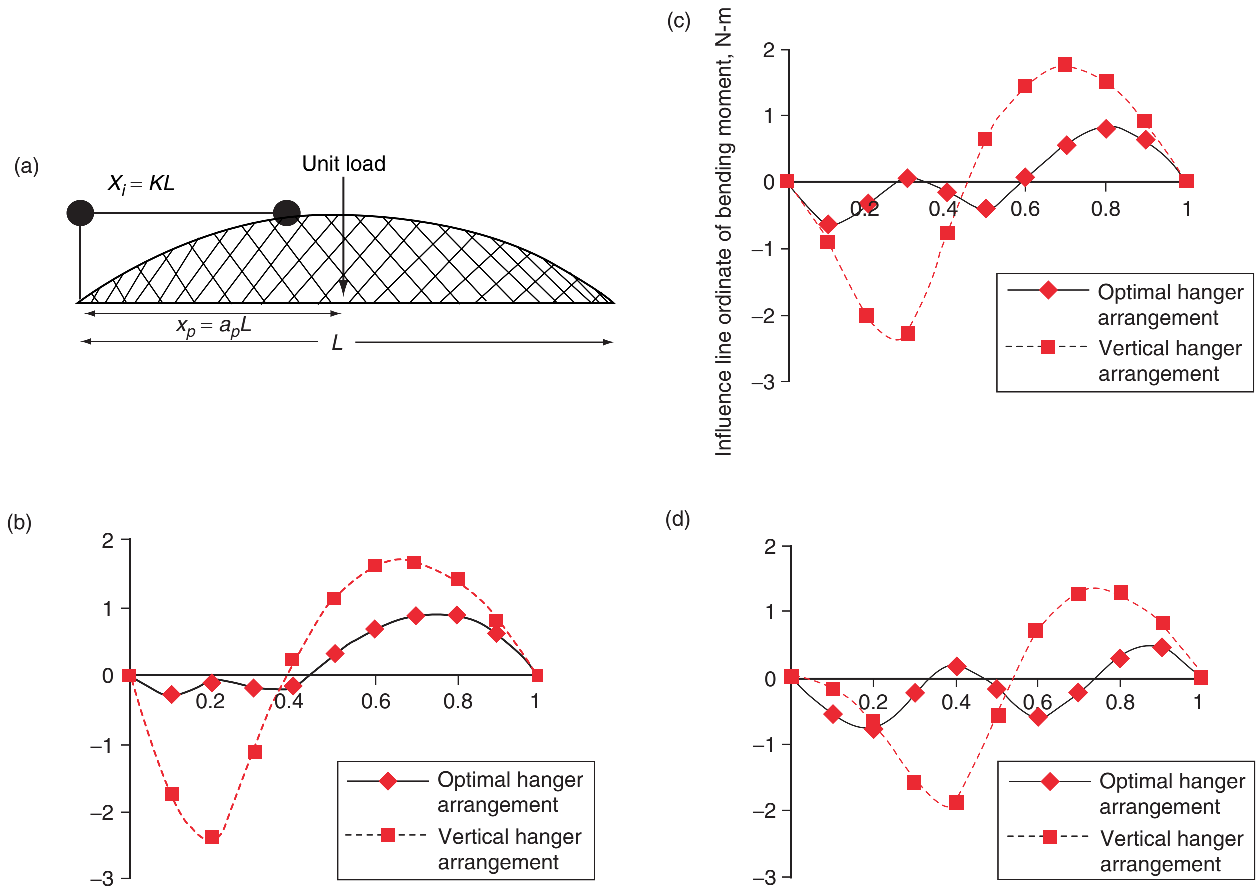

Figure 18. (a) Location of influence line along the arch; (b) Influence line for bending moment at $X = 0.2L$; (c) Influence line for bending moment at $X = 0.3L$; (d) Influence line for bending moment at $X = 0.4L$

arrangement divided by that ordinate for optimized hanger arrangement; K as shown in that figure is a factor for position of arch along the span, L such that for any distance of arch position X, $K = X/L$. The figure shows that at different arch position influence line ordinate (IL) for vertical hanger arrangement is approximately 2 to 40 times larger than that of optimized hanger arrangement.

Bending moments and reinforcement percentage required in the arch for specific load step are shown in Figure 20 which concludes that response parameters in the arch with optimized design variables are negligible

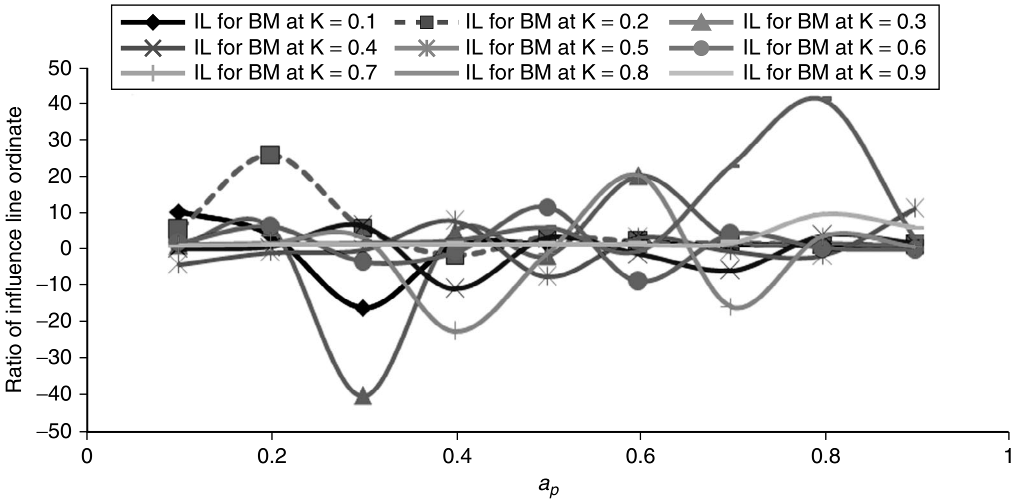

Figure 19. Ratio of influence line ordinate for bending moment at $X = KL$

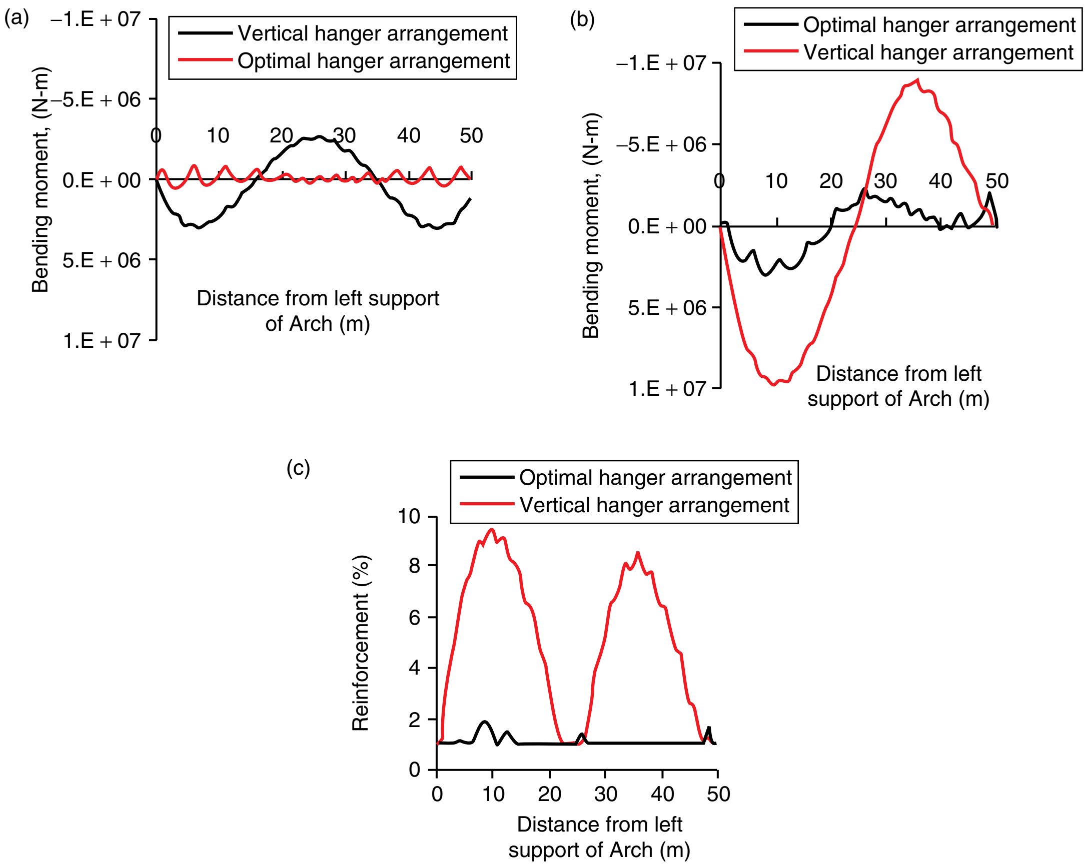

Figure 20. (a) Bending moment diagram of arch for dead load only; (b) Bending moment diagram of arch for a load combination of dead and live load; (c) Reinforcement percentage envelope of arch for a load combination of dead and live load

than that of arch with vertical hangers. Reinforcement percentage envelope required along the arch for all load steps is shown in Figure 21 which shows that vertical hanger arrangement requires larger amount of steel compared with optimized hanger arrangements. Results also show that maximum amount of steel is required at quarter span for vertical hanger arrangements.

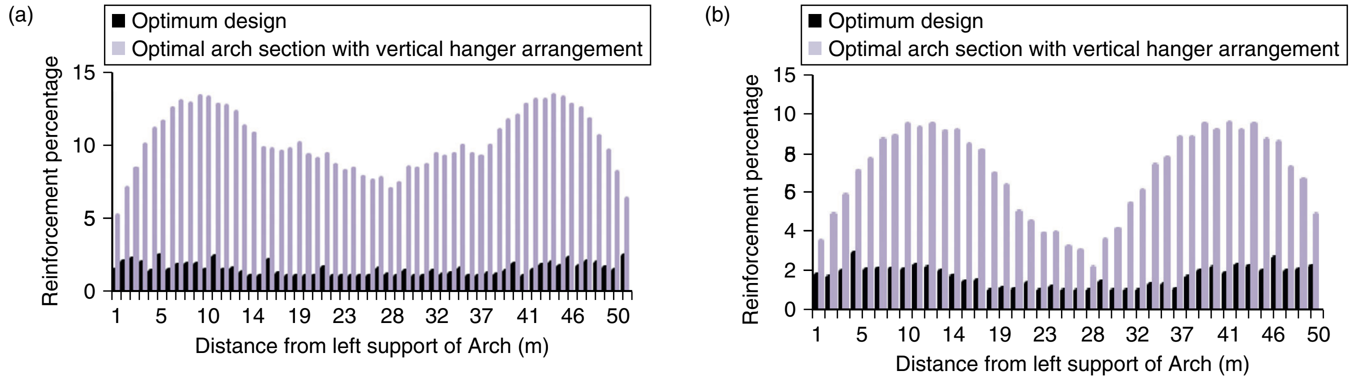

Figure 21. Reinforcement percentage envelope along the arch of: (a) circular arch geometry; (b) parabolic arch geometry

Table 6. Result comparison for cost of initial and optimized design

	Circular Arch		Parabolic Arch	
Cost Breakdown	**Initial**	**Final**	**Initial**	**Final**
Cost of Cable (BDT)	0.100001412E+07	0.199471517E+07	0.908678753E+06	0.154834904E+07
Cost of Concrete of Arch (BDT)	0.417691526E+07	0.180762830E+07	0.409068897E+07	0.176910130E+07
Cost of Reinforcement in Arch (BDT)	0.345129619E+07	0.134102902E+07	0.269647296E+07	0.145578683E+07
Total Cost (BDT)	0.862822557E+07	0.514337249E+07	0.769584069E+07	0.47650856E+07
		40.38% Save of Initial Design		38.1% Save of Initial Design

A cost breakdown for hanger, concrete of arch and arch reinforcement is outlined in Table 6 for initial and final optimized design which shows that 40.38% cost can be saved for circular arch and 38.1% cost can be saved for parabolic arch if design is optimized.

6. CONCLUSIONS

Based on the numerical analysis within the range of parameters examined, the following conclusions can be drawn for structural optimization of network arch bridges:

i) It is found that optimized hanger arrangement is attained for hangers placed at some certain inclinations with vertical. Hanger inclination is found same to all hangers for parabolic arch geometry whereas for circular arch geometry the inclination varies from one hanger to another in a small range. Under the scope of study, hanger inclination for parabolic arch geometry is found to be 32 degree whereas it varies from 36 to 39 degree from first hanger to the last for circular arch geometry.

ii) Parabolic arch geometry is found to be more economical than circular arch geometry considering optimum design.

iii) Under the scope of study, it is observed that optimized network arch bridge with circular arch geometry requires lesser number of hangers than parabolic arch geometry.

iv) Under the scope of study, it is found that optimized network arch bridge with circular arch geometry requires shallower arch than that required for parabolic arch geometry.

v) Within the range of design constant parameters it is observed that 36% to 40% of total cost can be saved for circular and parabolic arches if design is optimized.

Findings outlined here are limited for an arch bridge of particular span and lane. Therefore it is recommended that the study can be extended further in the following fields:

i) Optimization can be performed for different span and lane of bridge to have general conclusion on global optimum design of arch bridges.

ii) Dynamic properties of the bridge with optimum design variables may be investigated further.

iii) The study can be extended for Performance Based Optimality criteria as in Liang (2002).

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NOTATION

A_h	cross sectional area of cable of hanger
a, b, c, d	complex vertices
a_p, x_p	variable for defining unit load to determine influence line in arch
B_h	arch width
B_w	width of bridge
C	location of centroid of 'complex'
C_{AC}	cost of concrete
C_{AS}	cost of reinforcement
C_{HC}	cost of cable of hangers
C_T	total cost
CRT	strength criteria of arch
DL	dead load
E_C	modulus of elasticity for concrete
E_s	modulus of elasticity for steel
f_{ca}'	concrete compressive strength at 28 days for arch and bracing
f_{cd}'	concrete compressive strength at 28 days for deck
f_u	ultimate strength of cable of hanger
f_y	arch reinforcement yield stress
F_y	specified minimum yield stress of the type of steel being used
$f(x)$	objective function
f(a), f(b), f(c), f(d)	function values of complex vertices

$G(x)_i$	implicit values
$G(x)_i^L$	upper limit of implicit constraints
H_h	arch depth
k	number of vertices in complex of evop
X_i, K_i	variable for defining position of arch
L	span of arch
L_i, U_i	lower limit and upper limit symbol for constraints
N_h	total number of hangers
N_{h1}	number of hangers for set1 cable in each arch
NDIV	number of design variables
NIC	number of implicit constraints
R_h	rise to span ratio
RNR	reinforcement factor of the arch
S	length of element of arch
UP_{HC}	unit price of cable
UP_{AC}	unit price of concrete of arch
UP_{AS}	unit price of steel of arch
V_{AC}	volume of concrete
W_{AS}	weight of reinforcement
W_{HC}	weight of cables
X	the centroid of complex which lies in the infeasible area
x_k	design variables
x_k^L	lower limit of design variables
x_k^U	upper limit of design variables
$\sigma_{\max}$	extreme hanger stress
$\varphi 1$	start angle for set 1 cable
$\varphi 2$	start angle for set 2 cable
$\Delta\varphi 1$	angle change for set 1 cable
$\Delta\varphi 2$	angle change for set 2 cable
$\varphi \mathrm{T1}_n$	end angle for hanger inclination set1
$\varphi \mathrm{T2}_n$	end angle for hanger inclination set2
υ	Poisson's ratio of concrete
ρ	density
AASHTO	American association of state highway and transportation officials
AISC	American institute of steel construction
APDL	ANSYS parametric design language
ASTM	American society for testing and materials
EVOP	evolutionary operation
FEA	finite element analysis
LRFD	load and resistance factor design
RHD	roads and highway department