

Cost Optimum Design of Posttensioned I-Girder Bridge Using Global Optimization Algorithm

Raquib Ahsan¹; Shohel Rana²; and Sayeed Nurul Ghani³

Abstract: This paper presents an optimization approach to the design of simply supported, post-tensioned, prestressed concrete I-girder bridges. The objective is to minimize the total cost of the structure, considering cost of materials, fabrication, and installation. For a particular girder span and bridge width, the design variables considered for cost minimization of the bridge system are girder spacing, various cross-sectional dimensions of the girder, number of strands per tendon, number of tendons, tendon layout and configuration, slab thickness, slab rebar, and shear rebar for the girder. Explicit constraints on the design variables are developed on the basis of geometric requirements, practical conditions for construction, and code restrictions. Implicit constraints for design are formulated as per the American Association of State Highway and Transportation Officials (AASHTO) Standard Specifications. The optimization problem is characterized by having a combination of continuous, discrete, and integer sets of design variables and multiple local minima. An optimization algorithm, evolutionary operation (EVOP), is used that is capable of locating directly with high probability the global minimum without requiring information on gradient or subgradient of the objective function. The present optimization approach is used for a real-life bridge project, leading to a feasible and acceptable design resulting in around 35% savings in cost per square meter of the deck area. Computational time required for optimization of the present problem is only a few seconds. Because constant design parameters have influence on the optimum design, this cost minimization procedure is performed for a range of such parameters. DOI: 10.1061/(ASCE)ST.1943-541X.0000458. © 2012 American Society of Civil Engineers.

CE Database subject headings: Cost; Structural design; Girder bridges; Post tensioning; Optimization; Algorithms.

Author keywords: Cost optimum design; Prestressed concrete; Post-tensioned girder bridge; Constrained global optimization.

Introduction

Prestressed concrete (PC) I-girder bridge systems are ideal as short to medium span (20 to 60 m) highway bridges because of their moderate self weight, structural efficiency, ease of fabrication, fast construction, low initial cost, long life expectancy, low maintenance, simple deck removal, and replacement (Precast/Prestressed Concrete Institute (PCI) 2003). To compete with steel bridge systems, the design of PC I-girder bridges should lead to the most economical use of materials (PCI 1999). Large numbers of design variables are involved in the design process of the PC I-girder bridges, and all variables are related to one other, leading to numerous alternative feasible designs. In the traditional design approach, bridge engineers follow iterative procedures for designing prestressed I-girder bridge structures. There is no formal attempt to reach the best design in the strict mathematical sense of minimizing cost, weight, or volume. The design process relies solely on the designers' experience, intuition, and ingenuity resulting in a high cost in material, time, and human effort.

The optimum design procedure is an alternative to the traditional design approach. Such a design usually implies the most economic structure without impairing any of its functional purposes. The optimization technique transforms the conventional design process of trial and error to a formal, systematic, and digital computer-based automated procedure that yields a design that is the best in the designer-specified figure of merit—the goodness factor of design. Advances in numerical optimization methods, computer-based numerical tools for analysis, and design of structures and availability of powerful computing hardware have all significantly aided the design process. So, the time is now appropriate to perform research on realistically optimizing three-dimensional structures, especially large structures with hundreds of members where optimization can result in substantial savings (Adeli and Sarma 2006). Large and important projects containing I-girder bridge structures have the potential for substantial cost reduction through application of optimum design methodology and thus, will be of great value to practicing engineers.

Many research performed on structural optimization deal with minimization of the weight of the structure (Vanderplaats 1984; Arora 1989; Adeli and Kamal 1993; Adeli 1994; Cohn and Dinovitzer 1994; Adeli and Sarma 2006). For concrete structures, however, the approach to optimum design may take the form of cost minimization problem because different materials are involved. A review of articles pertaining to the cost optimization of concrete structures is presented by Sarma and Adeli (1998) and the same for concrete bridge structures by Hassanain and Loov (2003). Torres et al. (1966) and Fereig (1985, 1996) presented the minimum cost design of prestressed concrete bridges using the linear programming method. Cohn and MacRae (1984a, b) studied the minimum cost design of fully prestressed and partially prestressed concrete I-beams with fixed cross-sectional geometry using a

¹Professor, Dept. of Civil Engineering, Bangladesh Univ. of Engineering and Technology (BUET), Dhaka-1000, Bangladesh (corresponding author). E-mail: raquibahsan@ce.buet.ac.bd

²Assistant Professor, Dept. of Civil Engineering, Bangladesh Univ. of Engineering and Technology (BUET), Dhaka-1000, Bangladesh. E-mail: shohel@ce.buet.ac.bd

³Principal Engineer/Consultant, Optimum System Designers, 2387 E Skipping Rock Way, Tucson, AZ85737. E-mail: ghanisn@yahoo.com

Note. This manuscript was submitted on June 10, 2010; approved on June 15, 2011; published online on June 17, 2011. Discussion period open until July 1, 2012; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Structural Engineering*, Vol. 138, No. 2, February 1, 2012. ©ASCE, ISSN0733-9445/2012/2-273-284/\$25.00.

nonlinear programming technique. Jones (1985), Yu et al. (1986), and Fereig (1994) formulated a minimum cost design of the PC box girder bridge system. Lounis and Cohn (1993) presented a cost optimization method for short and medium span pretensioned PC I-girder bridge. Nonlinear programming methods (projected Lagrangian method and sieve-search technique) were utilized to obtain the minimum superstructure cost as a criterion for the best design. They first found the maximum feasible girder spacing for each standard Canadian Precast Concrete I (CPCI) girder and American Association of State Highway and Transportation Officials (AASHTO) sections and then minimized the prestressed and non-prestressed reinforcement in the I-girder and the deck. Costs of individual components, such as the girders or the deck, were then minimized. No attempt was made regarding ascertaining the optimal girder cross section and the spacing that may minimize the total cost inclusive of the cost of the bridge deck. Sirca and Adeli (2005) presented an optimization method for minimizing the total cost of the pretensioned PC I-beam bridge system by considering the concrete area, deck slab thickness, reinforcement, surface area of formwork, and number of beams as design variables. The problem is formulated as a mixed integer-discrete nonlinear programming problem and solved by using a patented robust neural dynamics model. They did not consider cross-sectional dimensions as design variables; instead, they used standard AASHTO sections. In the studies discussed previously, the prestressing strands/tendons were assumed to be located at a fixed eccentricity. In reality, however, strands/tendons are located at different positions at a cross section of the girder. The longitudinal profiles of tendons also vary depending on their location on a cross section. Again, a lump-sum value (a certain percentage of initial prestress) of prestress losses was estimated in these studies. As prestress losses are implicit functions of material properties, girder geometry and construction method, a more accurate assessment of these losses, however, is required for greater precision. Ayvaz and Aydin (2009) presented a study on minimizing the cost of a pretensioned PC I-girder bridge through topological and shape optimization. This topological and shape optimization of the bridge system were performed together with a genetic algorithm (GA).

In this paper, an optimum design approach for simply supported post-tensioned I-girder bridges with cross-sectional dimensions and tendon arrangements as design variables is developed, while considering the cost of materials, including fabrication and installation. The bridge system consists of precast girders with a cast-in situ reinforced concrete deck. A large number of design variables and constraints are considered, and an optimization algorithm evolutionary operation (EVOP) (Ghani 1989) is used. The algorithm is capable of dealing with an objective function containing a mix of integer, discrete, and continuous design variables and locating directly the global minimum with a high probability. The EVOP code is written in FORTRAN. The main program that executes EVOP is developed in C++ language to formulate the mathematical expressions required for analysis and design of the bridge system, to define a starting point inside the feasible space, and to input EVOP control parameters. Three functions are also defined in this program: an objective function, an explicit constraint function, and an implicit constraint function, all required by the EVOP code. All are then compiled, linked, and executed to perform a cost optimum design of the previously discussed bridge system. A case study is presented by comparing the optimum design obtained by the present approach with the design of a recently constructed structure. A parametric study is also conducted by optimizing the design of post-tensioned PC I-girder bridges for different sets of values of constant parameters.

Problem Formulation

Design Variables and Constant Design Parameters

For a particular girder span and bridge width, the design variables considered in this study are the spacing of girders, cross-sectional dimensions of a girder, number of strands per tendon, number of tendons, configuration of tendons, deck slab thickness, and deck slab reinforcement. The design variables and variable types are tabulated in Table 1. A typical cross section of the PC I-girder is illustrated in Fig. 1 to show several of these design variables. The constant design parameters under consideration are various

Table 1. Design Variables with Explicit Constraints

Design variables	Variable type	Explicit constraint
Girder spacing (S) (m)	Discrete	$B_W/10 \leq S \leq B_W$
Girder depth (G_d) (mm)	Discrete	$1,000 \leq G_d \leq 3,500$
Top flange width (TF _w) (mm)	Discrete	$300 \leq TF_w \leq S$
Top flange thickness (TF _t) (mm)	Discrete	$75 \leq TF_t \leq 300$
Top flange transition thickness (TFT _t) (mm)	Discrete	$50 \leq TFT_t \leq 300$
Bottom flange width (BF _w) (mm)	Discrete	$300 \leq BF_w \leq S$
Bottom flange thickness (BF _t) (mm)	Discrete	$a \leq BF_t \leq 600$
Web width (W _w) (mm)	Discrete	$b \leq W_w \leq 300$
Number of strands per tendon (N_s)	Integer	$1 \leq N_s \leq 27$
Number of tendons per girder (N_T)	Integer	$1 \leq N_T \leq 20$
Lowest tendon position at the end from bottom fiber (y_1) (mm)	Continuous	$A_M \leq y_1 \leq 1,000$
Initial stage prestress (% of full prestress) (η)	Continuous	$1\% \leq \eta \leq 100\%$
Slab thickness (t) (mm)	Discrete	$175 \leq t \leq 300$
Slab main reinforcement ratio (ρ)	Continuous	$\rho_{min} \leq \rho \leq \rho_{max}$

Note: B_W = bridge width; a = clear cover + duct diameter; b = clear cover + web rebars diameter + duct diameter; A_M = minimum vertical edge distance for anchorage; and ρ_{min} and ρ_{max} are, respectively, minimum and maximum permissible reinforcement of slab according to AASHTO (2002).

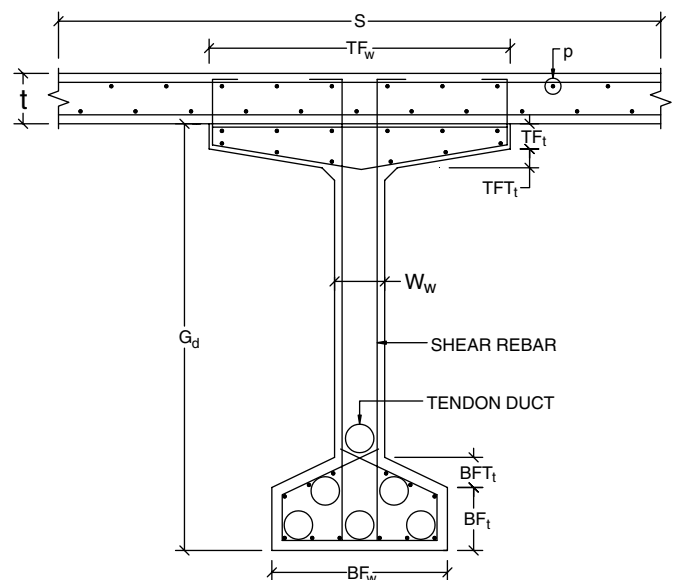


Fig. 1. Girder composite section with design variables

Table 2. Minimum Dimensions for *C* Range Anchorage System

Number of strands per tendon	1–3	4	5–7	8–9	10–12 (mm)	13	14–19	21–22	23–27
Duct diameter (mm)	45	50	65	70	85	85	100	110	115
Duct clear spacing (D_S) (mm)	38	38	38	38	38	38	38	50	50
A_D (mm)	110	120	150	185	200	210	250	275	300
A_M (mm)	128	150	188	210	248	255	300	323	345

Note: A_D = anchorage dimension.

material properties, superimposed dead loads, AASHTO live loads, strand size, post-tensioning anchorage system, and unit costs of materials, including fabrication and installation. The optimization is on the basis of the analysis of an interior girder, as shown in Fig. 2. The deck and the girders are assumed to act as a composite section during the service condition. It is considered that prestress is applied in two stages—initially a percentage of total prestress is applied to carry only the girder self weight and finally full prestress is applied during the casting of the deck slab. In the present study, the tendon arrangement is considered as variable because it has significant effects on prestress losses and flexural stress at various sections along the girder. Tendon layout along the span is assumed as parabolic. The vertical and horizontal arrangements of tendons depend on various cross-sectional dimensions of the girder, duct size and spacing, anchorage spacing, and anchorage edge distance. Only the Freyssinet *C* range anchorage system is considered in this study (Freyssinet 1999). Duct diameter, duct spacing, anchorage dimension, and anchorage spacing vary with the design variable of number of strands per tendon, as shown in Table 2. Typical arrangements of tendons at various sections are shown in Fig. 3.

Objective Function

In this study, the objective of design is cost minimization of a post-tensioned PC *I*-girder bridge system by taking into account costs of materials, fabrication, and installation. The total cost of the bridge system is determined as follows:

$$C_T = C_{GC} + C_{DC} + C_{PS} + C_{OS} \quad (1)$$

where C_{GC} , C_{DC} , C_{PS} and C_{OS} = cost of materials, fabrication, and installation of girder concrete, deck slab concrete, prestressing steel, and ordinary steel. Ordinary steel is used for deck reinforcement and the girder's shear reinforcement. Costs of individual components are calculated as follows:

$$C_{GC} = (UP_{GC}V_{GC} + UP_{GF}SA_G)N_G \quad (2)$$

$$C_{DC} = (UP_{DC}V_{DC} + UP_{DF}(S - TF_w))N_G \quad (3)$$

$$C_{PS} = (UP_{PS}W_{PS} + 2UP_{ANC}N_T + UP_{SH}N_TL)N_G \quad (4)$$

$$C_{OS} = UP_{OS}(W_{OSD} + W_{OSG})N_G \quad (5)$$

where UP_{GC} , UP_{DC} , UP_{PS} and UP_{OS} = unit prices, including materials, labor, fabrication, and installation of the precast girder concrete, deck concrete, prestressing steel, and ordinary steel respectively; UP_{GF} , UP_{DF} , UP_{ANC} , UP_{SH} = unit prices of girder formwork, deck formwork, anchorage set, and metal sheath for the duct, respectively; V_{GC} , V_{DC} , W_{PS} , W_{OSD} and W_{OSG} = volumes of the precast girder concrete and deck slab concrete, weight of the prestressing steel and ordinary steel in the deck and girder, respectively; L = girder span; N_G = number of girders; S = girder spacing; and SA_G = surface area of girder.

Explicit Constraints

These are specified limits (upper or lower limit) on design variables that are derived from geometric requirements (e.g., superstructure depth and clearances), minimum practical dimensions for construction, and code restrictions.

- Explicit constraints for the girder spacing: Lower and upper limits on girder spacing are considered such that the number of girders in the bridge can vary from 1 to 10.
- Explicit constraints for the top flange: The lower limit of the top flange width is assumed as 300 mm from lateral stability and bearing considerations, and the upper limit is equal to girder spacing. The lower limit of top flange thickness is considered 75 mm to resist damage during handling and to provide adequate space for proper placement of transverse reinforcement and the upper limit is assumed as 300 mm. The lower limit of top flange transition thickness is considered 50 mm to facilitate placement and consolidation of concrete, and the upper limit is assumed as 300 mm. The haunch thickness and width are both assumed as 50 mm.
- Explicit constraints for the web: The lower limit of the web width is equal to the sum of the diameter of a duct and diameter of the web rebars plus a clear cover. The upper limit is assumed as 300 mm.
- Explicit constraints for the bottom flange: The lower limit of bottom flange width is assumed as 300 mm to accommodate anchorage setup, and the upper limit is equal to the girder spacing. The lower limit of thickness is equal to the clear cover plus

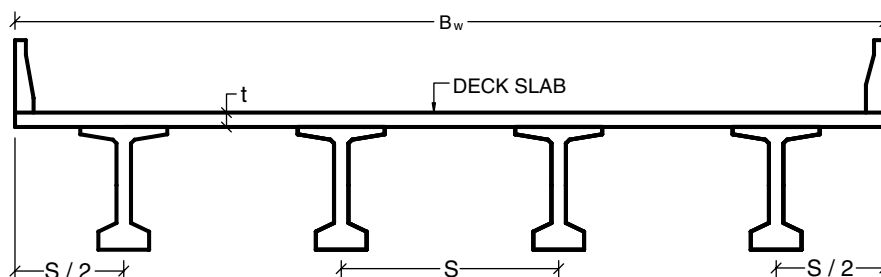


Fig. 2. Girders arrangement in the bridge

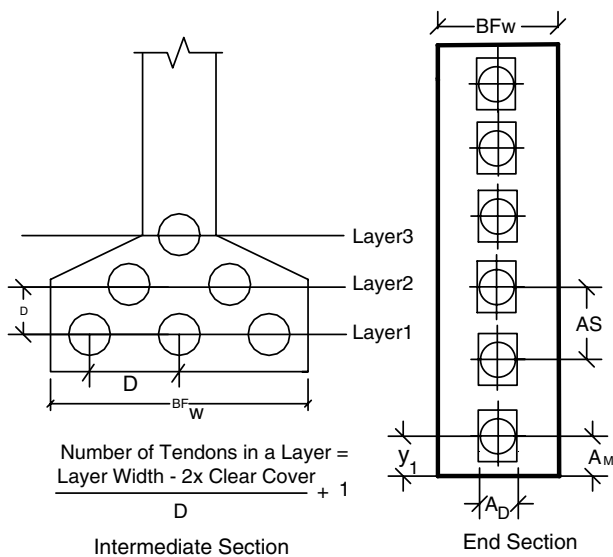


Fig. 3. Tendons arrangement in the girder

duct diameter to fit at least one row of tendons. The upper limit is assumed as 600 mm. The width to thickness ratio of the bottom flange transition area is assumed as 2 to 1 from a practical construction point of view.

- Explicit constraints for the girder depth: The lower limit of the girder depth is considered 1,000 mm and the upper limit is 3,500 mm, which is the common range of the girder depth to minimize the cost of substructure, approach roads, and aesthetics.
- Explicit constraints for the number of strands per tendon: Within the available anchorage system, one tendon may consist of several seven-wire strands, typically from 1 to 55. For the present study, it is considered that each tendon may consist of 1 to 27 strands.
- Explicit constraints for the number of tendons: The amount of prestressing force required for the cost optimum design is directly associated with the number of tendons required in the girder. For this study, it is considered that the number of tendons may vary from 1 to 20.
- Explicit constraints for the lowest tendon position: To vary the profile of the tendon along the girder span, the vertical position of the lowest tendon from the bottom fiber of the end section is considered a design variable. The vertical positions of other tendons at the end section are determined from anchorage spacing requirements. The lower limit of this design variable is the minimum permissible vertical edge distance of the C range

anchorage system shown in Table 2, and the upper limit is assumed as 1,000 mm.

- Explicit constraints for the deck slab: The lower limit of deck slab thickness is considered 175 mm to control deflection and excessive cracking, and the upper limit is 300 mm. The lower and upper limits of the deck slab reinforcement are considered according to AASHTO Standard Specification (AASHTO 2002). The explicit constraints for all the previous design variables are shown in Table 1.

Implicit Constraints

These constraints represent the performance or response requirements of the bridge system. A total 46 implicit constraints are considered according to the AASHTO (2002) and categorized in eight groups as follows:

1. flexural working stress constraints,
2. flexural ultimate strength constraints,
3. shear constraints (ultimate strength),
4. ductility constraints,
5. deflection constraint,
6. lateral stability constraint,
7. tendon eccentricity constraint, and
8. deck slab design constraint.

The constraints are formulated in subsequent sections.

Flexural Working Stress Constraints

These are the allowable stresses in concrete and are given as follows:

$$f^L \leq f_j \leq f^U \quad (6)$$

$$f_j = -\frac{F_j}{A} \pm \frac{F_j e_j}{S_j} \pm \frac{M_j}{S_j} \quad (7)$$

where f^L = allowable compressive stress (lower limit); f^U = allowable tensile stress (upper limit); f_j = the actual working stress in concrete; A = cross-sectional area of the girder; F_j , e_j , S_j , M_j = prestressing force, tendons eccentricity, section modulus, and working moment at the j th section, respectively. These constraints are considered at three critical sections along the span of the girder, as shown in Fig. 4, and for various loading stages (initial stage and service conditions). Loading stages and the corresponding implicit constraints are summarized in Table 3. The three critical sections are the mid section (Section 1), the section at the end of anchorage and transition zone (Section 2), and the section immediately after the anchor set in which the prestress is at its maximum value (Section 3). The end of anchorage and transition zone is assumed as 1.5 times the girder depth.

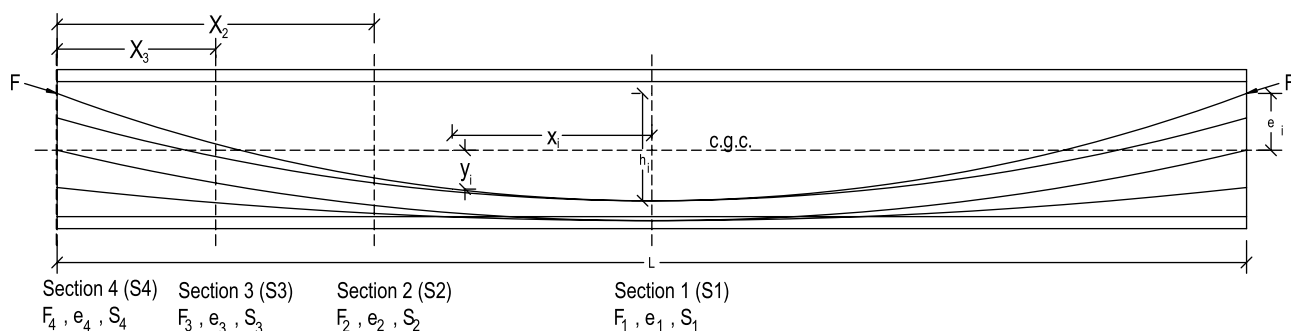


Fig. 4. Tendon's profile along the girder

Table 3. Loading Stages and Implicit Constraints

Loading stage	Resisting section	Section properties	Load combination	Implicit constraint
Initial stage	Precast section	A_{net}, e_i, S_{net}	$\eta F + G$	$-0.55f'_{ci} \leq f_j \leq 0.25\sqrt{f'_{ci}}$
1	Precast section	A_{jf}, e, S	$F_i + G + SB + DP$	$-0.60f'_c \leq f_j \leq 0.5\sqrt{f'_c}$
2	Precast section	A_{jf}, e, S	$F_e + G + SB + DP$	$-0.40f'_c \leq f_j \leq 0.5\sqrt{f'_c}$
	Composite section	S_C	+SD	
3	Precast section	A_{jf}, e, S	$F_e + G + SB + DP$	$-0.60f'_c \leq f_j \leq 0.5\sqrt{f'_c}$
	Composite section	S_C	+(SD + L + I)	
4	Precast section	A_{jf}, e, S	$0.5(F_e + DL)$	$-0.40f'_c \leq f_j \leq 0.5\sqrt{f'_c}$
	Composite section	S_C	+(L + I)	

Note: G = girder self weight; SB = slab weight; DP = diaphragm weight; SD = super-imposed dead load for wearing surface and curb weight; DL = total dead load; L = live load; I = impact load. A_{net} , A_{jf} = net area and transformed area of the precast girder; e_i , e = eccentricity of tendons at initial stage and at final stage of precast section, respectively; S_{net} , S , S_C = section modulus of net, transformed, and composite section of girder, respectively; f'_{ci} , f'_c = initial and 28 days compressive strengths of girder concrete, respectively.

Instead of using a lump-sum value, prestress losses are estimated according to AASHTO (2002) for greater accuracy because prestress losses are also implicit functions of some of the design variables. The instantaneous losses depend on the jacking equipment, and anchorage hardware used and the design variables i.e., the number of tendons, number of strands per tendon, layout of tendon in the girder, prestressing of tendon, and girder cross-sectional properties. The long-term losses are composed of loss attributed to the creep of concrete, loss attributed to the shrinkage of concrete, and loss attributed to steel relaxation. These losses are also implicit functions of concrete properties and girder cross-sectional properties. In the case of post-tensioning, prestressing force varies along the span of the girder. The prestressing forces after deduction of instantaneous losses at the various sections, as shown in Fig. 4, are determined as follows [Eqs. (8)–(11)]:

$$F_{1i} = \sum_{j=1}^{j=N_T} F [1 - e^{-(\mu\theta_j + K\frac{L}{2})}] - L_{ES} \quad (8)$$

$$F_{2i} = F - 0.5L_{ANC} - L_{ES} \quad (9)$$

$$F_{3i} = F_{2i} - 0.5 \left(\frac{x_2 - x_3}{x_2} \right) L_{ANC} - L_{ES} \quad (10)$$

$$F_{4i} = F - L_{ANC} - L_{ES} \quad (11)$$

where F_{1i} , F_{2i} , F_{3i} , F_{4i} = prestressing forces after instantaneous losses at various sections; F = jacking force; L_{ES} = elastic shortening loss; L_{ANC} = anchorage loss; K and μ are wobble and friction coefficients, respectively; and θ_j = initial angle at the end section of the j th tendon.

The prestress forces after all loss deduction at the three sections are denoted by F_{1e} , F_{2e} , and F_{3e} , respectively. As per AASHTO (2002), for post-tensioned members, allowable prestressing force for the tendon immediately after seating at anchorage is $0.7f_{pu}A_s$, at the end of the seating loss zone it is $0.83f_y^*A_s$, and that at the service load after all loss deduction is $0.80f_y^*A_s$. In the present study, jacking force is considered equal to $0.9f_y^*A_s$ and then three constraints [Eq. (12)–(14)] are applied to keep the forces in the tendon within the allowable limits.

$$0 \leq F_{4i} \leq 0.7f_{pu}A_s \quad (12)$$

$$0 \leq F_{2i} \leq 0.83f_y^*A_s \quad (13)$$

$$0 \leq F_{2e} \leq 0.80f_y^*A_s \quad (14)$$

where total prestressing steel area, $A_s = A_{strand}N_sN_T$; A_{strand} = prestressing steel area per strand; f_{pu} , f_y^* = ultimate strength and yield stress of prestressing steel, respectively; and f_y = yield stress of ordinary steel.

Ultimate Flexural Strength Constraints

The ultimate flexural strength constraints for the precast and composite sections are as follows:

$$0 \leq M_{pu} \leq \varphi M_{pn} \quad (15)$$

$$0 \leq M_{cu} \leq \varphi M_{cn} \quad (16)$$

where M_{pu} and M_{cu} = factored bending moments; and φM_{pn} and φM_{cn} = flexural strengths of the precast and composite sections, respectively. To calculate the flexural strength of the composite section the following four cases are considered:

Case 1: Compression block remains within the deck slab;

Case 2: Compression block remains within the top flange;

Case 3: Compression block remains within the top flange transition area; and

Case 4: Compression block falls in web (flanged section calculation is used assuming T shape stress block).

Ductility (Maximum and Minimum Prestressing Steel) Constraints

The maximum prestressing steel constraint for the composite section is given as follows:

$$0 \leq \omega \leq \omega_u \quad (17)$$

where ω = reinforcement index; and ω_u = upper limit of reinforcement index.

The constraint that limits the minimum value of reinforcement is

$$1.2M_{cr}^* \leq \varphi M_n \quad (18)$$

where M_{cr}^* , φM_n = cracking moment and ultimate moment at a critical section, respectively.

Ultimate and Horizontal Shear Strength Constraints

The ultimate shear strength is considered at two sections, the section at the end of the transition zone and the section where the prestress is maximum. The related implicit constraint is defined as follows:

$$\varphi V_s = (V_u - \varphi V_c) \leq 0.67\sqrt{f'_c}W_w d_s \quad (19)$$

where V_u = factored shear at a section; V_c = the concrete contribution taken as lesser of flexural shear, V_{ci} and web shear, V_{cw} , in kilonewtons; and V_s = shear carried by the steel in kN. These two shear capacities are determined according to AASHTO (2002).

The constraint for horizontal shear for composite section is

$$V_u \leq \varphi V_{nh} \quad (20)$$

where V_{nh} = nominal horizontal shear strength.

Deflection Constraints

Deflection attributed to live load (AISC Marketing 1986) is

$$\Delta_{LL} = \frac{324}{E_c I_c} P_T (L^3 - 555L + 4780) \quad (21)$$

where P_T = weight of one front wheel multiplied by the distribution factor plus impact; E_c = modulus of elasticity of girder concrete; and I_c = moment of inertia of the composite girder section.

The live load deflection constraint is

$$\Delta_{LL} \leq \frac{L}{800} \quad (22)$$

End Section Tendon Eccentricity Constraint

The constraint that limits the tendon eccentricity at the end section so that the extreme fiber tension remains within the allowable limits is as follows:

$$\frac{G_d}{6} + 0.25 \sqrt{f'_{ci}} \frac{A_{end} G_d}{6F_{4i}} \leq e_4 \leq \frac{G_d}{6} + 0.5 \sqrt{f'_c} \frac{A_{end} G_d}{6F_{4e}} \quad (23)$$

where A_{end} = cross-sectional area of the end section of the girder.

Lateral Stability Constraint

The following constraint, according to PCI (2003), ensures the safety and stability during the lifting of a long girder, subject to roll about the weak axis

$$FS_c \geq 1.5 \quad (24)$$

where FS_c = factor of safety against cracking of top flange when the girder hangs from the lifting loop.

Deck Slab Constraints

The constraint considered for deck slab thickness according to the design criteria of ODOT (2000) is

$$t \geq \frac{S_d + 17}{3} \quad (25)$$

The constraint that limits the required effective depth for deck slab is

$$d_{min} \leq d_{req} \leq d_{prov} \quad (26)$$

where S_d = effective slab span in feet = $S - TF_w/2$; t = slab thickness in inches; and d_{req} , d_{prov} , d_{min} = required, provided and minimum effective depth of deck slab, respectively.

Optimization Method

The bridge optimization problem under discussion is characterized by a large number of design variables and constraints. The design variables are a combination of continuous, discrete, and integer types. Expressions for the objective function and the constraints

are nonlinear functions of these design variables. The optimal design problem, therefore, becomes highly nonlinear and nonconvex, having multiple local minimums that require a method capable of locating the global optimum by handling mixed integer, discrete, and continuous arguments.

The procedure EVOP has successfully minimized a large number of recognized test problems (Ghani 1989, 1995). An updated version of EVOP is used in this study that is capable of minimizing objective functions having a combination of integer, discrete, and continuous arguments as well. The method treats all arguments as continuous, but for discrete and integer design variables, the method picks values from within thin strips centered on specified values. A Users' Manual for EVOP with several test examples can be found in Ghani (2008).

The algorithm can minimize an objective function

$$f_0(x) = f_0(x_1, x_2, \dots, x_n) \quad (27)$$

where $f_0(x)$ = function of n independent variables $x^T = (x_1, x_2, \dots, x_n)$.

The n independent variables x_i 's ($i = 1, 2, \dots, n$) are subject to explicit constraints

$$l_i \leq x_i \leq u_i \quad (28)$$

where l_i 's and u_i 's = lower and upper limits on the variables, respectively. They are either constants or functions of n independent variables (moving boundaries). These explicit constraints must define, at best, a convex or, at worst, a semiconvex n -dimensional vector-space but never a nonconvex vector-space. Any explicit constraint that renders the vector-space nonconvex should be classified as an implicit constraint discussed subsequently and shifted to the implicit constraint set. The corresponding variable, however, should be given a fixed upper and a lower limit and included in the explicit constraint set, thus, rendering the vector-space defined by only the explicit constraints to become semiconvex.

These n independent variables x_i 's are also subject to m numbers of implicit constraints

$$L_j \leq f_j(x_1, x_2, \dots, x_n) \leq U_j \quad (29)$$

where $j = 1, 2, \dots, m$; and L_j 's and U_j 's = lower and upper limits on the m implicit constraints, respectively. They are either constants or functions of the n independent variables. The implicit constraints are allowed to make the feasible vector-space nonconvex.

Additionally, the optimization may also be subject to k numbers of equality constraints of the form

$$h_j(x) = 0 \quad \text{where } j = 1, 2, \dots, k \quad (30)$$

Functions $h_j(x)$ are also of arbitrary complexity of n independent variables (x_1, x_2, \dots, x_n) . The procedure is unable to directly deal with the previous form of equality constraints. It, however, can indirectly handle such equality constraints by defining an augmented objective function

$$F(x, \lambda) = f_0(x) + \sum \lambda_j h_j(x) \quad (31)$$

where $j = 1, \dots, k$; and λ_j 's = unknown weighting factors such that all $h_j(x)$ vanish at the minimum of $F(x, \lambda)$.

In the literature, λ_j 's are known as Lagrange multipliers.

The previous problem needs to be presented to the subroutine EVOP as follows:

$$\text{Minimize } F(x, \lambda) = f_0(x) + \sum \lambda_j h_j(x) \quad \text{where } j = 1, \dots, k \quad (32)$$

Subject to explicit constraints $l_i \leq x_i \leq u_i, i = 1, \dots, n$ Eq. (28) repeated

$$\text{and } 0 \leq \lambda_j \leq \text{any positive number } j = 1, \dots, k \quad (33)$$

and an additional set of k numbers of implicit constraints

$$0 \leq h_j(x) \leq \text{any positive number greater than } h_j(x^0) \quad (34)$$

where x^0 = starting point.

If the numerical value of $h_j(x)$ is negative, then either negate $h_j(x)$ to make it positive or just square it.

Procedure

The method is subdivided into six fundamental processes, which are described in Ghani (1989). They are, (1) generation of a complex, (2) selection of a complex vertex for penalization, (3) testing for collapse of a complex, (4) dealing with a collapsed complex, (5) movement of a complex, and (6) convergence tests (Fig. 5).

A complex is a living object spanning an n -dimensional parameter space defined by $k \geq (n + 1)$ vertices inside the feasible

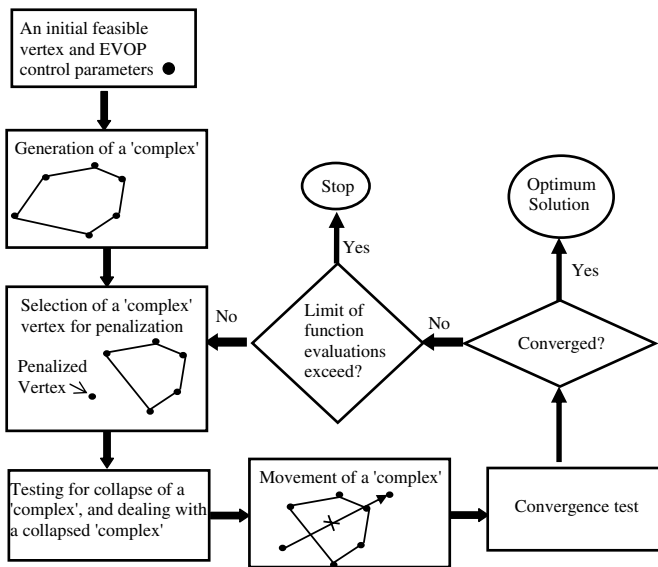


Fig. 5. General outline of EVOP Algorithm

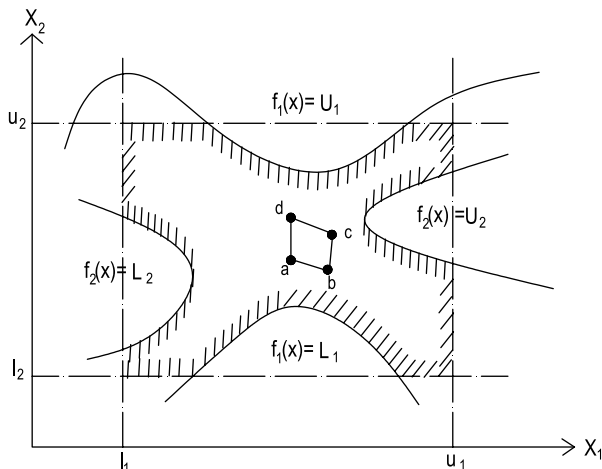


Fig. 6. A complex with four vertices

region. It has the intelligence to move toward a minimum located on the boundary or inside the allowed space. It can rapidly change its shape and size for negotiating difficult terrain. Fig. 6 shows a complex with four vertices in a two-dimensional parameter space. The complex vertices are identified by lower case letters 'a', 'b', 'c' and 'd' in an ascending order of function values, i.e., $f(a) < f(b) < f(c) < f(d)$. A straight line parallel to the coordinate axes are explicit constraints with fixed upper and lower limits. The curved lines represent implicit constraints set to either upper or lower limits. The hatched area is the two-dimensional feasible search spaces.

Generating a Feasible Starting Point

Generating a feasible starting point is simple. Before calling subroutine EVOP, a random point satisfying all explicit constraints is generated in one step by using Eq. (28). The coordinates of this random point is given by:

$$x_i = l_i + r_i(u_i - l_i) \quad \text{where } (i = 1, 2 \dots n) \quad (35)$$

where r_i = pseudorandom deviate of rectangular distribution over the interval (0,1).

For this to be possible, the problem has to be so defined that all explicit constraints have constant upper and lower bounds. Those explicit constraints having variable upper or lower bounds are shifted from the list of explicit constraints and included within the set of implicit constraints. Satisfaction of all implicit constraints is next ascertained, and if any violation is detected this randomly generated point is rejected and another test point satisfying all explicit constraints is created as discussed previously. Once again satisfaction of all implicit constraints is verified. When a test point is found that has succeeded in satisfying all explicit and implicit constraints, subroutine EVOP is called and the results of optimization scrutinized.

Case Study

In this section, an example is provided to demonstrate the practical application of the approach presented in this paper. Present method is applied to a real-life bridge project. The bridge, 750-m long and 12.11-m wide, is to be built in the northern part of Bangladesh. The optimum design obtained here is compared with the existing design of the project. The bridge is a prestressed concrete I girder bridge of medium span (50 m) made composite with cast-in situ deck slab (BRTC 2007). The constant design parameters used are summarized in Table 4. The cost data for materials, labor, fabrication, and installation used for the optimum design are identical to that for the existing design, as obtained from the Roads and Highway Department cost schedule (RHD 2006). Comparative values of the design variables, cost of the existing design, and the cost of the optimized design are presented in Table 5. The cross sections showing the design variables corresponding to the existing design and the optimum design of the bridge are given in Figs. 7 and 8, respectively. In Table 5, the minimum cost design produces an optimal I-girder bridge system configuration that would be 35% more economical than the existing design. There are significant differences in almost all of the design variables between the two. Girder spacing is greater in the optimum design, resulting in a lesser number of girders for the best design than the existing design (Figs. 7 and 8). In the most economical design, the girder depth, top flange width, bottom flange thickness, and slab thickness, are comparatively bigger and the top flange thickness, bottom flange width, web width, prestressing steel, and deck slab reinforcement are smaller than the existing design. Arrangements of tendons of the existing design and the optimum design are shown in Figs. 9 and 10, respectively. In the

Table 4. Constant Design Parameters

Cost data	Material properties	Bridge design data
$UP_{GC} = \$180$ per m^3	$f_{pu} = 1,861$ MPa	Girder length = 50 m ($L = 48.8$ m)
$UP_{GF} = \$5$ per m^2	$f_y = 410$ MPa	Bridge width, $B_w = 12.0$ m (3 lane)
$UP_{DC} = \$85$ per m^3	$f'_c = 40$ MPa	Live load = HS20-44 (both truck loading and lane loading)
$UP_{DF} = \$4.5$ per m^2	$f'_{cd} = 25$ MPa	No of diaphragm = 4; diaphragm width = 250 mm
$UP_{PS} = \$1,285$ per t	$f'_{ci} = 30$ MPa	Wearing surface = 50 mm
$UP_{ANC} = \$65$ per set	$K = 0.005/m$	Curb height = 600 mm; curb width = 450 mm
$UP_{SH} = \$1.3$ per linear m	$\mu = 0.25$	7 wire low-relaxation strand
$UP_{OS} = \$640$ per t	$\delta = 6$ mm	Freyssinet anchorage system

Note: f'_{cd} = compressive strength of slab concrete.

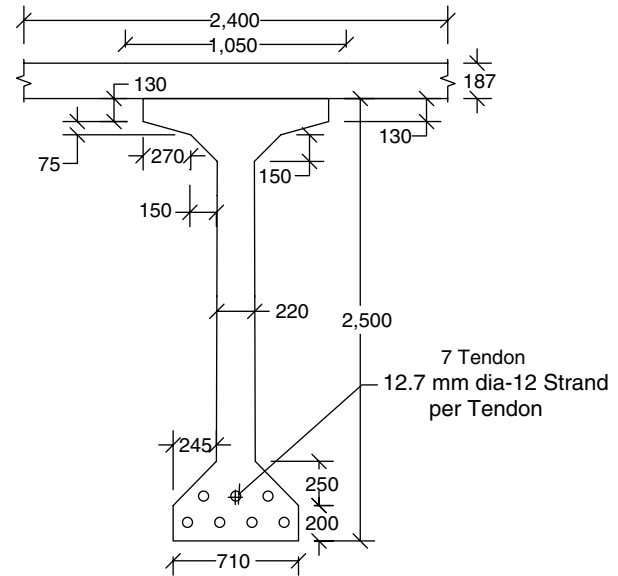
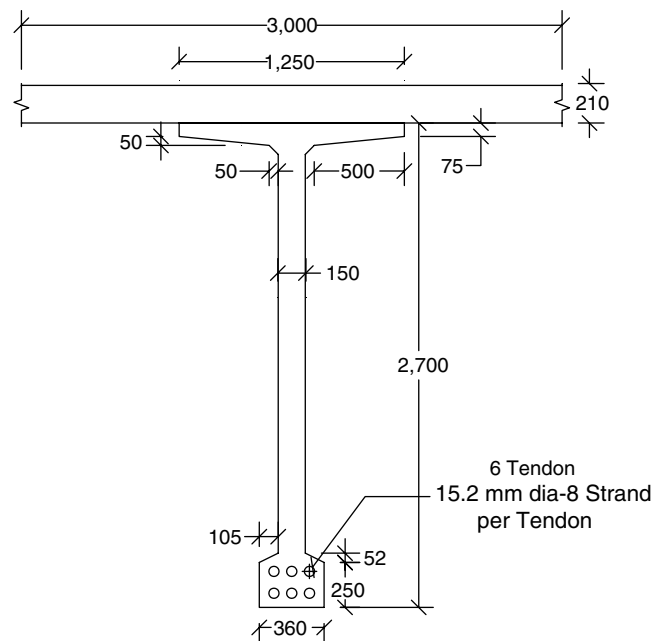
Table 5. Existing Design and Cost Optimum Design

Design Variables	Existing design	Optimum design
Girder spacing (S) (m)	2.4	3.0
Girder depth (G_d) (mm)	2,500	2,700
Top flange width (TF _w) (mm)	1,060	1,250
Top flange thickness (TF _t) (mm)	130	75
Top flange transition thickness (mm)	75	50
Web width (W_w) (mm)	220	150
Bottom flange width (BF _w) (mm)	710	360
Bottom flange thickness (BF _t) (mm)	200	250
Number of strands per tendon (N_s)	12 (0.5" dia)	8 (0.6" dia)
Number of tendons per girder (N_T)	7	6
Lowest tendon position (y_1)	400	930
Initial stage prestress (η)	42.8%	53%
Slab thickness (t) (mm)	187.5	210
Slab main reinforcement ratio (ρ)	0.82%	0.63%
Other cross-sectional parameters	Existing design	Optimum design
Top flange transition width (mm)	270	500
Top flange haunch width (mm)	150	50
Top flange haunch thickness (mm)	150	50
Bottom flange transition width (mm)	245	105
Bottom flange transition thickness (mm)	250	52.5
Total cost per square meter of deck (\$)	175	113
%SAVING = $[(175 - 113)/175 \times 100]$		35.0%

optimum design, tendon arrangement is steeper than that of the existing design. This indicates that consideration of tendon arrangement as a design variable is important because it affects, to a great extent, the prestress losses and flexural stress at various sections along the girder. Web reinforcement of the girder is also less in the optimum design than that of the existing design (Figs. 11 and 12). The optimization problem with 14 mixed type design variables and 46 implicit constraints converges with just 32 numbers of objective function evaluations with 3 digit accuracy. The corresponding numbers of explicit and implicit constraints evaluations are 256 and 171, respectively. An Intel COREi3 processor has been used in this study, and computational time required for optimization by EVOP is around only 2 s. Further, the design process becomes fully automated, making costly expert human involvement unnecessary.

Parametric Study

The present cost optimum design method has been applied for 3 lane and 4 lane bridges of a 50 m girder span. The AASHTO

**Fig. 7.** The existing design (dimensions are in millimeters)**Fig. 8.** The optimum design obtained in this study (dimensions are in millimeters)

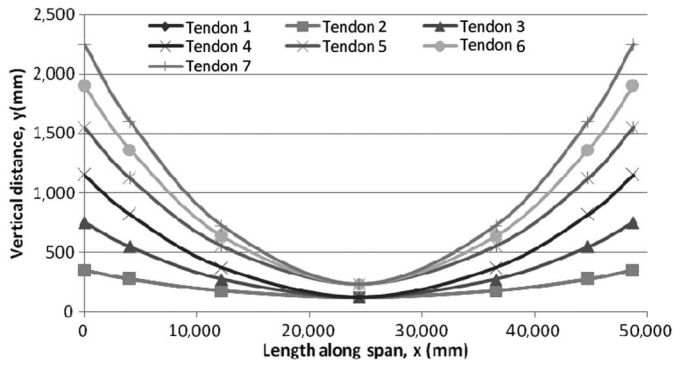


Fig. 9. Tendon profile in the existing design

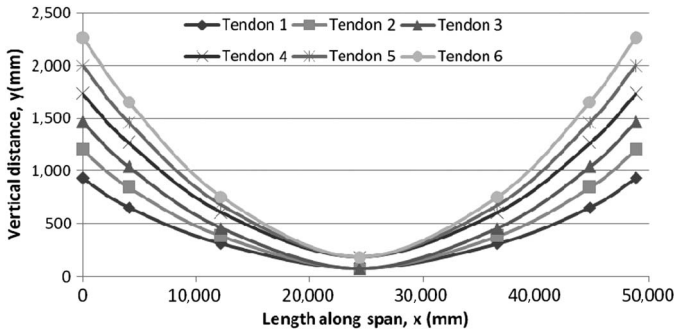


Fig. 10. Tendon profile in the optimum design

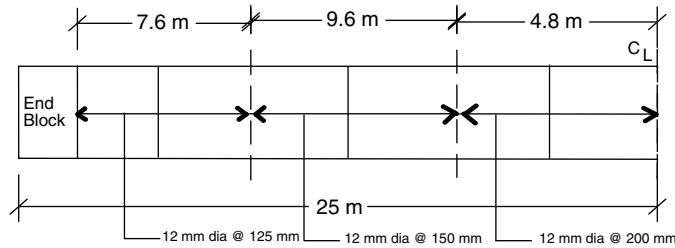


Fig. 11. Web reinforcement of girder in the existing design

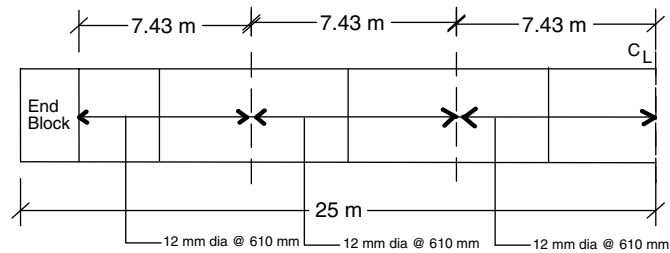


Fig. 12. Web reinforcement of girder in the optimum design

HS20-44 live load (both truck loading and lane loading) with superimposed dead loads, as per AASHTO (2002), has been considered for each case. The optimum design is sensitive to the design constant parameters i.e., unit cost of materials, labor, fabrication and installation, concrete strength, strand size, and anchorage system. Different design constant parameters result in different optimum designs. The effect of two girder concrete strengths i.e., 40 and 50 MPa (28 days) are studied. Three different regimes of unit costs of the materials, as shown in Table 6, are considered so that the

variation in design with cost can be observed. Concrete strength at the initial stage is taken as 75% of 28 days strength. Deck slab concrete strength is considered 25 MPa. Ultimate strength of prestressing steel and yield strength of ordinary steel are considered 1,861 and 410 MPa, respectively. The Freyssinet C-Range anchorage system is used for post-tensioning tendons consisting of 15.2-mm diameter 7-wire strands. The unit cost of girder concrete and deck slab concrete are considered fixed. The unit costs of steel and anchorage systems are varied such that in Cost 2, the cost is twice those in Cost 1, and in Cost 3, the cost is three times those in Cost 1.

From the cost optimum design study for the present bridge system, the following observations are made:

1. Optimum girder spacing for a 3 lane bridge is 3 m for both concrete strengths and for a 4 lane bridge is 3.2 m for all the cases, irrespective of the difference in costs of materials (Tables 7–9). Optimum girder depth for Cost 1 is 2,540 mm (3 lanes) and 2,640 mm (4 lanes) for 50 MPa concrete strength and 2,700 mm (3 lanes) for 40 MPa concrete strength. Girder depth increases with the increase of the cost of steel in both cases, which indicates that the relative cost difference of materials influences the optimum design of bridge.
2. Because of composite construction, deck slab thickness is usually adequate to satisfy the compression area required for flexural strength of the girder. The top flange width is controlled by the effective span of the deck slab to satisfy the serviceability criteria of the deck and lateral stability effects of the girder. Top flange width increases in Cost 2 but does not increase for Cost 3. Optimum top flange width is in between 1,050 to 1,200 mm in the case of concrete strength of 50 MPa and 1,100 to 1,400 mm in the case of concrete strength of 40 MPa. This indicates that the wider top flange reduces the formwork cost of the deck slab and increases the safety factor against lateral stability. The optimum top flange thickness and top flange transition thickness, however, stick to their lower limit.
3. The optimum bottom flange width is approximately 370 mm for both concrete strengths, which is close to its lower limit. This indicates that it is not necessary to have a large bottom flange width to accommodate all the tendons in the lowermost position to have greater eccentricity. Thus, the bottom flange transition area is minimized to keep the concrete area smaller. Bottom flange thickness increases a little with the increase in the cost of steel for Cost 2 but decreases for Cost 3.
4. Optimum web width in all the three cases is 150 mm, and the number of strands per tendon is 8 or 9 for both concrete strengths. This indicates that the C-Range anchorage system, which accommodates 9 tendons, is the optimum value (Tables 7 and 8) for the range of variables considered here. The number of tendons required decreases with the increase in the cost of steel.
5. With an increasing steel cost, the deck slab thickness increases only a little because larger thickness induces a greater dead load. Therefore, the optimum value of deck slab thickness is 210 to 220 mm, irrespective of the rise in steel cost. However, the reinforcement ratio of the deck decreases with an escalating steel price.
6. The percentage of steel to be prestressed at initial stage increases with an increase in steel cost. This indicates that as steel cost increases, the girder weight also increases, requiring more prestress at the initial stage. In this study, the variable tendon arrangement along the girder is considered. Vertical positions of tendons at various sections are shown in Table 10. With the increase in the cost of steel, the tendons are arranged in the girder such that vertical distances of tendons decrease

Table 6. Cost Regimes Used for Cost Minimum Design

Item	Unit	Cost 1 (\$) (C1)	Cost 2 (\$) (C2)	Cost 3 (\$) (C3)
Precast girder concrete-including equipment and labor (UP _{GC})	per m ³ (50 MPa)	250	250	250
	per m ³ (40 MPa)	180	180	180
Girder formwork (UP _{GF})	per m ²	5	5	5
Cast-in-place deck concrete (UP _{DC})	per m ³	85	85	85
Deck formwork-equipment and labor (UP _{DF})	per m ²	4.5	4.5	4.5
Girder post-tensioning-tendon, equipment, and labor (UP _{PS})	per ton	1,285	2,570	3,850
Anchorage set (UP _{ANC})	per set	65	130	195
Metal sheath for duct (UP _{SH})	per linear meter	1.3	2.6	3.9
Mild steel reinforcement for deck and web in girder (UP _{OS})	per ton	640	1,280	1,920

Table 7. Optimum Values of Design Variables for 3 Lane 50-m Girder and Concrete Strength = 50 MPa

Cost	S (m)	G _d (mm)	TF _w (mm)	TF _t (mm)	TFS _t (mm)	BF _w (mm)	BF _t (mm)	W _w (mm)	N _S	N _T	t (mm)	ρ%	y ₁ (mm)	η%
C1	3.0	2,540	1,050	75	50	375	215	150	9	6	215	0.62	770	53
C2	3.0	2,780	1,175	75	50	370	250	150	9	5	215	0.59	890	66
C3	3.0	3,100	1,075	75	50	365	180	150	8	5	220	0.57	760	70

Table 8. Optimum Values of Design Variables for 3 Lane 50-m Girder and Concrete Strength = 40 MPa

Cost	S (m)	G _d (mm)	TF _w (mm)	TF _t (mm)	TFS _t (mm)	BF _w (mm)	BF _t (mm)	W _w (mm)	N _S	N _T	t (mm)	ρ%	y ₁ (mm)	η%
C1	3.0	2,700	1,250	75	50	360	250	150	8	6	210	0.63	930	53
C2	3.0	3,020	1,375	75	50	380	280	150	8	5	215	0.57	830	56
C3	3.0	3,460	1,125	75	50	320	165	150	9	4	220	0.55	895	70

Table 9. Values of Design Variables for 4 Lane 50-m Girder and Concrete Strength = 50 MPa

	S (m)	G _d (mm)	TF _w (mm)	TF _t (mm)	TFS _t (mm)	BF _w (mm)	BF _t (mm)	W _w (mm)	N _S	N _T	t (mm)	ρ%	y ₁ (mm)	η%	TC* (\$) (C3)
C1	3.2	2,640	1,050	75	50	365	250	150	9	6	215	0.82	843	43	127
C2	3.2	2,800	1,150	75	50	380	200	150	9	5	220	0.63	707	47	170
C3	3.2	2,970	1,175	75	50	380	230	150	9	5	230	0.56	950	48	211

Table 10. Center of Gravity of Tendons from Bottom Fiber of Girder in the Optimum Design

	Girder concrete strength = 50 MPa					Girder concrete strength = 40 MPa				
	Y ₁	Y ₂	Y ₃	Y _{end}	AS	Y ₁	Y ₂	Y ₃	Y _{end}	AS
C1	127	440	1,010	1,366	235	127	510	1,152	1,600	268
C2	116	433	973	1,363	238	116	428	930	1,345	256
C3	116	384	819	1,190	213	127	455	923	1,412	346

Note: AS = anchorage spacing.

from the bottom of the girder, resulting in an increase of eccentricity (Table 10).

- With a two-fold increase in the unit cost of steel, for the case of concrete strength of 50 MPa, the optimized cost of girder decreases by \$7.00 per m² of deck slab area, resulting in a 4% savings in cost. When the cost of steel increases by three times, the optimized cost of the girder decreases by \$16.00 per m² of deck slab, ensuing around 8% savings. For 40 MPa, the savings are approximately 4.5% for Cost 2 and 10% for Cost 3 (Table 11).
- The most active constraints governing the optimum design are compressive stress at the top fiber of the girder for permanent

dead load at the service condition, tensile stress at bottom fiber attributed to all loads, prestress force at the end of seating loss zone, deck thickness, factors of safety against lateral stability, and deflection at the service condition attributed to a full load in most of the three cases. As the amount of prestressing of steel decreases with increasing cost of steel, the flexural strength of the composite girder becomes an important constraint.

- Control parameters used and computational effort required by EVOP are tabulated in Table 12. It shows that the optimization problem with a large number of mixed type design variables and implicit constraints converges with a rather small number

Table 11. Cost of Individual Materials

Cost (\$)	Girder concrete strength = 50 MPa					Girder concrete strength = 40 MPa				
	C_{GC}	C_{DC}	C_{PS}	C_{OS}	TC	C_{GC}	C_{DC}	C_{PS}	C_{OS}	TC
C1	52	21	34	18	125	44	20	31	18	113
C2	58	21	55	36	170	49	20	50	38	157
C3	62	21	78	52	213	51	22	65	54	192

Note: Cost in U.S. dollars per square meter of deck slab; TC = total cost.

Table 12. Computational Effort and Control Parameters Used

	Girder concrete strength = 50 MPa										Girder concrete strength = 40 MPa									
	OF	EC	IC	T(s)	α	β	γ	Φ	Φ_{cpx}		OF	EC	IC	T(s)	α	β	γ	Φ	Φ_{cpx}	
C1	95	639	389	3	1.2	0.5	2	10^{-13}	10^{-16}		32	256	171	2	1.3	0.5	2	10^{-13}	10^{-16}	
C2	85	478	333	3	1.5	0.5	2	10^{-13}	10^{-16}		50	298	309	2	1.6	0.5	2	10^{-13}	10^{-16}	
C3	63	402	272	3	1.7	0.5	2	10^{-13}	10^{-16}		72	371	279	2	1.2	0.5	2	10^{-13}	10^{-16}	

Note: Number of evaluations; OF = objective function; EC = explicit constraint; IC = implicit constraint; T = time (s); α , β , γ , Φ , Φ_{cpx} are EVOP control parameters.

of function evaluations. It may be useful to compare the previous figures with many staff-hours of effort by highly skilled engineering staff power needed for manual iterations.

Conclusions and Recommendations

The most economical design of a simply supported post-tensioned prestressed concrete *I*-girder bridge system is presented. A global optimization algorithm, EVOP, is used in this study. It is capable of locating directly with high probability the global minimum of an objective function of several variables and of arbitrary complexity, subject to explicit and implicit constraints. A digital computer program is developed that may be useful to designers and contractors interested in cost optimization of the *I*-girder bridge system. The influence of constant design parameters, such as unit costs of materials and concrete strength on the optimum design, is studied. The proposed cost optimum design approach is applied to a real-life project and shows considerable (35%) savings in cost, while computing a feasible and acceptable optimum design. Conclusions from this study are listed subsequently.

For all the cases studied, the optimum girder spacing remains identical to 3 m for a 3 lane bridge and 3.2 m for a 4 lane bridge. The optimum girder depth increases with the increase in the cost of steel. Optimum top flange width is controlled by the effective deck slab span and lateral stability effects of the girder. Optimum top flange thickness and top flange transition thickness are equal to their lower limits. Optimum bottom flange width remains close to the lower limit. Optimum web width remains nearly constant, irrespective of girder span and concrete strength. The optimum number of strands per tendon is 8 or 9 for 40 and 50 MPa concrete strengths. With the increase in steel cost, deck thickness does not increase comparatively and remains within 210 to 220 mm.

The relative cost difference of materials influences the optimum design of a bridge. For example, the cost optimum design when performed for a different relative cost of materials resulted in a new optimum design that saved 4 to 10% more of the overall cost of the bridge than the original design. The optimum cost of the bridge per square meter of the deck remained almost the same, irrespective of the number of the lanes.

It is difficult to solve the present constrained global optimization problem of 14 numbers of mixed integers, discrete and continuous design variables, and a large number of implicit constraints by using gradient based techniques. Such a problem can, however, be

easily dealt by EVOP with a relatively small number of function evaluations.

It is recommended that the optimization study be further extended for a continuous *I*-Girder bridge system or other types of bridge systems considering both superstructure and substructure and also for high strength concrete (HSC) girders. It will be also interesting to conduct such studies for modern synthetic materials, such as plastic composites. Smart materials for structures that can self repair (Yachuan and Jinping 2008) are gradually appearing on the scene. Applicability and economics of such advanced material for bridge construction may also be investigated. So far, we have discussed the optimization of a single or scalar objective function—the cost of the bridge. It is our intention to extend this work to cover multiple or vector objective functions.

The bottom line of this work is that in today's highly competitive world, diligence is simply not good enough for survival; one has to perform intelligently and through optimization.

References

- AASHTO. (2002). *Standard specifications for highway bridges*, 17th Ed., Washington, DC.
- Adeli, H., ed. (1994). *Advances in design optimization*, Chapman and Hall, London.
- Adeli, H., and Kamal, O. (1993). *Parallel processing in structural engineering*, Elsevier Applied Science, London.
- Adeli, H., and Sarma, K. C. (2006). *Cost optimization of structures*, Wiley, Chichester, UK.
- American Institute of Steel Construction (AISC) Marketing, Inc. (1986). *Highway structures design handbook*, Vols. I and II, Pittsburgh, PA.
- Arora, J. S. (1989). *Introduction to optimum design*, McGraw-Hill, New York.
- Ayvas, Y., and Aydin, Z. (2009). "Optimum topology and shape design of prestressed concrete bridge girders using a genetic algorithm." *Struct. Multi. Optim.*, 41(1), 151–162.
- Bureau of Research Testing & Consultation (BRTC). (2007). "Teesta bridge project report." *File No. 1247*, Dept. of Civil Engineering Library, Bangladesh Univ. of Engineering and Technology, Dhaka, Bangladesh.
- Cohn, M. Z., and Dinovitzer, A. S. (1994). "Application of structural optimization." *J. Struct. Eng.*, 120(2), 617–650.
- Cohn, M. Z., and MacRae, A. J. (1984a). "Optimization of structural concrete beams." *J. Struct. Eng.*, 110(7), 1573–1588.
- Cohn, M. Z., and MacRae, A. J. (1984b). "Prestressing optimization and its implications for design." *PCI J.*, 29(4), 68–83.

- Fereig, S. M. (1985). "Preliminary design of standard CPCI prestressed bridge girders by linear programming." *Can. J. Civ. Eng.*, 12(1), 213–225.
- Fereig, S. M. (1994). "An application of linear programming to bridge design with standard prestressed girders." *Comput. Struct.*, 50(4), 455–469.
- Fereig, S. M. (1996). "Economic preliminary design of bridges with prestressed I-girders." *J. Bridge Eng.*, 1(1), 18–25.
- Freyssinet Inc. (1999). "The C Range post-tensioning system." (www.freyssinet.com) (May 10, 2010).
- Ghani, S. N. (1989). "A versatile algorithm for optimization of a nonlinear non-differentiable constrained objective function." *UKAEA Harwell Rep. No. R-13714*, HMSO Publications Centre, London.
- Ghani, S. N. (1995). "Performance of global optimisation algorithm EVOP for non-linear non-differentiable constrained objective functions." *Proc., IEEE Int. Conf. on Evolutionary Computation*, Vol. 1, IEEE, New York, 320–325.
- Ghani, S. N. (2008). "User's guide to subroutine EVOP." (<http://www.OptimumSystemDesigners.com>) (Dec. 17, 2008).
- Hassanain, M. A., and Loov, R. E. (2003). "Cost optimization of concrete bridge infrastructure." *Can. J. Civ. Eng.*, 30(5), 841–849.
- Jones, H. L. (1985). "Minimum cost prestressed concrete beam design." *J. Struct. Eng.*, 111(11), 2464–2478.
- Lounis, Z., and Cohn, M. Z. (1993). "Optimization of precast prestressed concrete bridge girder systems." *PCI J.*, 38(4), 60–78.
- Ohio Dept. of Transportation (ODOT). (2000). *Bridge design manual*, Columbus, OH.
- Precast/Prestressed Concrete Institute (PCI). (1999). *PCI design handbook*, 5th Ed., Chicago.
- Precast/Prestressed Concrete Institute (PCI). (2003). *PCI bridge design manual*, Chicago.
- Roads and Highway Department (RHD). (2006). *Schedule of rates*, Dhaka, Bangladesh.
- Sarma, K. C., and Adeli, H. (1998). "Cost optimization of concrete structures." *J. Struct. Eng.*, 124(5), 570–579.
- Sirca, G. F., and Adeli, H. (2005). "Cost optimization of prestressed concrete bridges." *J. Struct. Eng.*, 131(3), 380–388.
- Torres, G. G. B., Brotchie, J. F., and Cornell, C. A. (1966). "A program for the optimum design of prestressed concrete highway bridges." *PCI J.*, 11(3), 63–71.
- Vanderplaats, G. N. (1984). *Numerical optimization techniques for engineering design with applications*, McGraw-Hill, New York.
- Yachuan, K., and Jinping, O. (2008). "Self-repairing performance of concrete beams strengthened using superelastic SMA wires in combination with adhesives released from hollow fibers." *Smart Mater. Struct.*, 17(2), 025020.
- Yu, C. H., Das Gupta, N. C., and Paul, H. (1986). "Optimization of prestressed concrete bridge girders." *Eng. Optim.*, 10(1), 13–24.